

# Dynamics & Statics

Previous year Questions  
from 2020 to 1992

2021-22

# 2020

1. A uniform rod, in vertical position, can turn freely about one of its ends and is pulled aside from the vertical by a horizontal force acting at the other end of pulled aside from the vertical by a horizontal force acting at the other end of the rod and equal to half its weight. At what inclination to the vertical will the rod rest? [10 Marks]
2. A light rigid rod  $ABC$  has three particles each of mass  $m$  attached to it at  $A, B$  and  $C$  the rod is struck by a blow  $P$  at right angles to it at a point distance from  $A$  equal to  $BC$ . Prove that the kinetic energy set up is  $\frac{1}{2} \frac{P^2}{m} \frac{a^2 - ab + b^2}{a^2 + ab + b^2}$  where  $AB = a$  and  $BC = b$  [10 Marks]
3. A beam  $AD$  rests on two supports  $B$  and  $C$ , where  $AB = BC = CD$ . It is found that the beam will tilt when a weight of is hung from or when a weight of  $p$  kg is hung from  $A$  or when a weight of  $q$  kg is hung from  $D$ . Find the weight of the beam. [15 Marks]
4. A square framework formed of uniform heavy rods of equal weight  $W$  jointed together, is hung up by one corner. A weight  $W$  is suspended from each of the three lower corners, and the shape of the square is preserved by a light rod along the horizontal diagonal. Find the thrust of the light rod. [10 Marks]
5. A particle starts at a great distance with velocity  $V$ . let  $p$  be the length of the perpendicular from the centre of a star on the tangent to the initial path of the particle. Show that the least distance of the particle from the centre of the star is  $\lambda$ , where  $V^2 \lambda = \sqrt{\mu^2 + p^2 V^4} - \mu$ . Here  $\mu$  is a constant. [10 Marks]
6. A four-wheeled railway truck has a total mass  $M$ , the mass and radius of gyration of each pair of wheels and axel  $m$  are  $k$  respectively, and the radius of each wheel is  $r$ . Prove that if the truck is propelled along a level track by a force  $P$ , the acceleration is  $\frac{P}{M + \frac{2mk^2}{r^2}}$ , and find the horizontal force exerted on each axel by the truck. The axle friction and wind resistance are to be neglected. [15 Marks]

# 2019

7. One end of a heavy uniform rod  $AB$  can slide along a rough horizontal rod  $AC$  to which it is attached by a ring.  $B$  and  $C$  are joined by a string. When the rod is on the point of sliding then  $AC^2 - AB^2 = BC^2$ . If  $\theta$  is the angle between  $AB$  and the horizontal line, then prove that the coefficient of friction is  $\frac{\cot \theta}{2 + \cot^2 \theta}$  [10 Marks]
8. The force of attraction of a particle by the earth is inversely proportional to the square of its distance from the earth's centre. A Particle, whose weight on the surface of the earth is  $W$ , falls to the surface of the earth from a height  $3h$  above it. Show that the magnitude of work done by the earth's attraction force is  $\frac{3}{4} hW$ , where  $h$  is the radius of the earth. [10 Marks]
9. A body consists of cone and underlying hemisphere. The base of the cone and the top of the hemisphere have same radius  $a$ . The whole-body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is  $\sqrt{3}a$  [15 Marks]
10. A particle moving along the  $y$ -axis has an acceleration  $Fy$  towards the origin, where  $F$  is a positive and even function of  $y$ . The periodic time, when the particle vibrated between  $y = -a$  and  $y = a$  is  $T$ . Show that  $\frac{2\pi}{\sqrt{F_1}} \leq T \leq \frac{2\pi}{\sqrt{F_2}}$  where  $F_1$  and  $F_2$  are the greatest and the least values of  $F$  within the range  $[-a, a]$ . Further show that when a simple pendulum of length  $l$  oscillates through  $30^\circ$  on either side of the vertical line,  $T$  lies between  $2\pi\sqrt{l/g}$  and  $2\pi\sqrt{l/g}, \sqrt{\pi/3}$  [20 Marks]

11. Prove that the path of a planet, which is moving so that its acceleration is always directed to a fixed point (star) and is equal to  $\frac{\mu}{(\text{distance})^2}$ , is a conic section. Find the conditions under which the path becomes (i) ellipse (ii) parabola and (iii) hyperbola. [15 Marks]

## 2018

12. A particle projected from a given point on the ground just clears a wall of height  $h$  at a distance  $d$  from the point of projection. If the particle moves in a vertical plane and if the horizontal range is  $R$  find elevation of the projection. [10 Marks]
13. A particle moving with simple harmonic motion in a straight line has velocities  $v_1$  and  $v_2$  at distances  $x_1$  and  $x_2$  respectively from the centre of its path. Find the period of its motion. [12 Marks]

## 2017

14. Suppose that the streamlines of the fluid flow are given by a family of curves  $xy = c$ . Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines. [10 Marks]
15. A fixed wire is in the shape of the Cardioid  $r = a(1 + \cos \theta)$ , the initial line being the downward vertical. A small ring of mass  $m$  can slide on the wire and is attached to the point  $r = 0$  of the Cardioid by an elastic string of natural length  $a$  and modulus of elasticity  $4mg$ . The string is released from rest when the string is horizontal. Show by using the laws of conservation of energy that  $a\theta^2(1 + \cos \theta) - g \cos \theta(1 - \cos \theta) = 0$ ,  $g$  being the acceleration due to gravity. [10 Marks]
16. A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle  $\phi$  with its curve surface touching the plane. Find the greatest admissible value of the inclination  $\phi$  for equilibrium. If  $\phi$  be less than this value, is the equilibrium stable? [17 Marks]
17. A particle is free to move on a smooth vertical circular wire of radius  $a$ . At time  $t = 0$  is projected along the circle from its lowest point  $A$  with velocity just sufficient to carry it to the highest point  $B$ . Find the time  $T$  at which the reaction between the particle and the wire is zero. [17 Marks]
18. A spherical shot of  $W$  gm weight and radius  $r$  cm, lies at the bottom of cylindrical bucket of radius  $R$  cm. The bucket is filled with water up to a depth of  $h$  cm ( $h > 2r$ ). Show that the minimum amount of work done in lifting the shot just clear of the water must be  $\left[ W \left( h - \frac{4r^3}{3R^2} \right) + W' \left( r - h + \frac{4r^3}{3R^2} \right) \right]$  cm gm.  $W'$  gm is the weight of water displaced by the shot. [16 Marks]

## 2016

19. A particle moves with a central acceleration which varies inversely as the cube of the distance. If it is projected from an apse at a distance  $a$  from the origin with a velocity which  $\sqrt{2}$  is times the velocity for a circle of radius  $a$  then find the equation to the path. [10 Marks]
20. A uniform rod  $AB$  of length  $2a$  movable about a hinge at  $A$  rests with other end against a smooth vertical wall. If  $\alpha$  is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is  $\frac{1}{2} W \sqrt{4 + \tan^2 \alpha}$  where  $W$  is the weight of the rod. [15 Marks]

21. Two weights  $P$  and  $Q$  are suspended from a fixed point  $O$  by strings  $OA, OB$  and are kept apart by a light rod  $AB$  if the strings  $OA$  and  $OB$  make angle  $\alpha$  and  $\beta$  with the rod  $AB$  show that the angle  $\theta$  which the rod makes with the vertical is given by  $\theta = \frac{P + Q}{P \cot \alpha - Q \cot \beta}$  [15 Marks]
22. A square  $ABCD$  the length of whose side is  $a$  is fixed in a vertical plane with two of its sides horizontal. An endless string of length  $l$  ( $> 4a$ ) passes over four pegs at the angle of the board and through a ring of weight  $W$  which is hanging vertically show that the tension of the string is  $\frac{W(l - 3a)}{(\sqrt{l^2 - 6la + 8a^2})}$ . [20 Marks]
23. A particle moves in a straight line. Its acceleration is directed towards a fixed point  $O$  in the line and is always equal to  $\mu \left( \frac{a^5}{x^2} \right)^{1/3}$  when it is at a distance  $x$  from  $O$ . If it starts rest at a distance  $a$  from  $O$ , then find the time the particle will arrive at  $O$ . [15 marks]

## 2015

24. A body moving under SHM has an amplitude  $a$  and time period  $T$ , If velocity is trebled, when the distance from mean position is  $\frac{2}{3}a$ , the period being unaltered find the new amplitude. [10 Marks]
25. A rod of  $8\text{kg}$  is movable in a vertical plane about a hinge at one end another end is fastened a weight equal to half of the rod, this is fastened by a string of length  $l$  to a point at a height  $h$  above the hinge vertically. Obtain the tension in the string. [10 Marks]
26. Two equal ladders of weight  $4\text{kg}$  each are placed so as to lean at  $A$  against each other with their end resting on a rough floor, given the coefficient of friction is  $\mu$ . The ladders at  $A$  make an angle  $60^\circ$  with each other. Find what weight on the top would cause them to slip. [13 Marks]
27. A mass  $m$  starts from rest at a distance ' $a$ ' from the center of force which attract inversely as the distance find the time of arriving at the center. [13 Marks]
28. A particle is projected from the base of a hill whose slope is that of a right circular cone whose axis is vertical. The projectile grazes the vertex and strikes the hill again at point on the base. if the semi vertical angle of the cone is  $30^\circ$   $h$  is height determine the initial velocity  $u$  of the projection and its angle of projection. [13 Marks]
29. Find the length of an endless chain which will hang over circular pulley of radius ' $a$ ' so as to be in contact with the two thirds of the circumference of the pulley. [12 Marks]
30. A particle moves in a plane a force towards a fixed centre proportional to the distance. If the path of the particle has apsidal distance  $a, b$  ( $a > b$ ), then find the equation of the path. [13 Marks]

## 2014

31. A particle is performing a simple harmonic motion (S.H.M) of period  $T$  about a centre  $O$  with amplitude  $a$  and it passes through a points  $P$  where  $OP = b$  in the direction  $OP$ . Prove that the time which elapse before it returns to  $P$  is  $\frac{T}{\pi} \cos^{-1} \left( \frac{b}{a} \right)$ . [10 Marks]
32. Two equal uniform rods  $AB$  and  $AC$ , each of length  $l$  are freely jointed at  $A$  and rest on a smooth fixed vertical circle of radius  $r$ . If  $2\theta$  is the angle between the rods, then find the relation between  $l, r$  and  $\theta$  by using the principal of virtual work. [10 Marks]

33. A particle of mass  $m$ , hanging vertically from a fixed point by a light inextensible cord of length  $l$  is struck by a horizontal blow which imparts to it a velocity  $2\sqrt{gl}$ . Find the velocity and height of the particle from its initial position when the cord becomes slack. **[15 Marks]**
34. A regular pentagon  $ABCDE$ , formed of equal heavy uniform bars jointed together is suspended from the joint  $A$ , and is maintained in form by a light rod joining the middle points of  $BC$  and  $DE$ . Find the stress in this rod. **[20 Marks]**
35. A particle is acted on by a force parallel to the axis of  $y$  whose acceleration (always towards the axis of  $x$ ) is  $\mu y^{-2}$  and when  $y = a$  it is projected parallel to the axis of  $x$  with velocity  $\sqrt{\frac{2\mu}{a}}$ . Find the parametric equation on the path of the particle. Here  $\mu$  is constant. **[15 Marks]**

## 2013

36. A body is performing S.H.M in a straight line  $OPQ$ . Its velocity is zero at points  $P$  and  $Q$  whose distances from  $O$  are  $x$  and  $y$  respectively and its velocity is  $v$  at the mid-point between  $P$  and  $Q$ . Find the time of one complete oscillation. **[10 Marks]**
37. The base of an inclined plane is 4 metres in length and the height is 3 metres. A force of 8kg acting parallel to the plane will just prevent a weight of 20 kg from sliding down. Find the coefficient of friction between the plane and the weight. **[10 Marks]**
38. A particle of mass 2.5 kg hangs at the end of a string 0.9 m long, the other end of which is attached to a fixed point. The particle is projected horizontally with a velocity 8 m/sec. Find the velocity of the particle and tension in the string when the string is (i) horizontal (ii) vertically upward. **[20 Marks]**
39. A uniform ladder rests at an angle of  $45^\circ$  with the horizontal with its upper extremity against a rough vertical wall and its lower extremity on the ground. If  $\mu$  and  $\mu'$  are the coefficients of limiting friction between the ladder and the ground and wall respectively, find the minimum horizontal force required to move the lower end of the ladder towards the wall. **[15 Marks]**
40. Six equal rods  $AB, BC, CD, DE, EF$  and  $FA$  are each of weight  $w$  and are freely jointed at their extremities so as to form a hexagon; the rod  $AB$  is fixed in a horizontal position and the middle points of  $AB$  and  $DE$  are joined by string. Find the tension in the string. **[15 Marks]**

## 2012

41. A particle moves with an acceleration  $\mu \left( x + \frac{a^4}{x^3} \right)$  towards the origin. If it starts from rest at a distance  $a$  from the origin, find its velocity when its distance from the origin is  $\frac{a}{2}$ . **[12 Marks]**
42. A heavy ring of mass  $m$ , slides on a smooth vertical rod and is attached to a light string which passes over a small pulley distance  $a$  from the rod and has a mass  $M (> m)$  fastened to its other end. Show that if the ring be dropped from a point in the rod in the same horizontal plane as the pulley it will descend a distance  $\frac{2Mma}{M^2 - m^2}$  before coming to rest. **[20 Marks]**
43. A heavy hemispherical shell of radius  $a$  has a particle attached at a point on the rim, and rests with the curved surface in contact with a rough sphere of radius  $b$  at the highest point. Prove that if  $\frac{b}{a} > \sqrt{5} - 1$  the equilibrium is stable, whatever be the weight of the particle. **[20 Marks]**

44. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is  $\mu \log \left[ \frac{1 + \sqrt{1 + \mu^2}}{\mu} \right]$ , where  $\mu$  the coefficient of friction is.

[20 Marks]

## 2011

45. The velocity of a train increases from 0 to  $v$  at constant acceleration  $f_1$  then remains constant for an interval and again decreases to 0 at a constant retardation  $f_2$ . If the total distance described is  $x$  find the total time taken. [10 Marks]
46. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a meter short of it when the angle of projection is  $\alpha$  and goes  $y$  meter beyond  $w$  then the angle of projection is  $\beta$ . If the velocity of projection is assumed same in all cases, find the correct angle of projection. [10 Marks]
47. A mass of 560kg. moving with a velocity of 240 m/ sec strikes a fixed target and is brought to rest in  $\frac{1}{100}$  sec. Find the impulse of the blow on the target and assuming the resistance to be uniform throughout the time taken by the body in coming to rest, find the distance through which it penetrates. [20 Marks]
48. A ladder of weight  $W$  rests with one end against a smooth vertical wall and the other end rest on a smooth floor. If the inclination of the ladder to the horizon is  $60^\circ$ , find the horizontal force that they must be applied to the lower end to prevent the ladder from slipping down. [20 Marks]
49. After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half its velocity if it now reaches the ground in 1 second, find the height of glass above the ground. [10 Marks]
50. A particle of mass  $m$  moves on straight line under an attractive force  $mn^2x$  towards a point  $O$  on the line, where  $x$  is the distance from  $O$ . If  $x = a$  and  $\frac{dx}{dt} = u$  when  $t = 0$ , find  $x(t)$  for any time  $t > 0$  [10 Marks]

## 2010

51. If  $v_1, v_2, v_3$  are the velocities at three points  $A, B, C$  of the path of a projectile, where the inclinations to the horizon are  $\alpha, \alpha - \beta, \alpha - 2\beta$  and if  $t_1, t_2$  are the times of describing the arcs  $AB, BC$  respectively. Prove that  $v_3 t_1 = v_1 t_2$  and  $\frac{1}{v_1} + \frac{1}{v_3} = \frac{2 \cos \beta}{v_2}$  [12 Marks]
52. A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. [20 Marks]
53. A particle moves with a central acceleration  $\mu(r^5 - 9r)$  being projected from an apse at a distance  $\sqrt{3}$  with velocity  $3\sqrt{2\mu}$ . Show that the path is the curve  $x^4 + y^4 = 9$ . [20 Marks]
54. Solid hemisphere is supported by a string fixed to point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If  $\theta$  and  $\phi$  are the inclination of the string and the plane base of the hemisphere to the vertical, prove by using the principal of virtual work that  $\tan \phi = \frac{3}{8} + \tan \theta$ . [20 Marks]

## 2009

55. A body is describing an ellipse of eccentricity  $e$  under the action of a central force directed towards a focus and when at the nearer apse, the center of force is transferred to the other focus. Find the eccentricity of the new orbit in terms of the eccentricity of the original orbit. [12 Marks]

56. A uniform rod  $AB$  is movable about a hinge at  $A$  and rests with one end in contact with a smooth vertical wall. If the rod is inclined at an angle of  $30^\circ$  with the horizontal, find the reaction at the hinge in magnitude and direction. **[12 Marks]**
57. A shot fired with a velocity  $V$  at an elevation  $\alpha$  strikes a point  $P$  in a horizontal plane through the point of projection. If the point  $P$  is receding from the gun with velocity  $v$ , show that the elevation must be changed to  $\theta$  where  $\sin 2\theta = \sin 2\alpha + \frac{2v}{V} \sin \theta$ . **[12 Marks]**
58. One end of light elastic of natural length  $l$  and modulus of elasticity  $2mg$  is attached to fixed point  $O$  and the other end to a particle of mass  $m$ , the particle initially held at rest at  $O$  is let fall. Find the greatest extension of the string during the motion and show that the particle will reach  $O$  again after a time. **[20 Marks]**
- $$\left(\pi + 2 - \tan^{-1} 2\right) \sqrt{\frac{2l}{g}}$$
59. A particle is projected with velocity  $V$  from the cusp of a smooth inverted cycloid down an arc. Show that the time of reaching the vertex is  $2\sqrt{\frac{a}{g}} \cot^{-1} \left(\frac{V}{2\sqrt{ag}}\right)$  **[10 Marks]**
60. Find the length of an endless chain which will hang over a circular pulley of radius  $a$  so as to be in contact with three-fourth of the circumference of the pulley. **[15 Marks]**

## 2008

61. A smooth parabolic tube is placed with vertex downwards in vertical plane. A particle slides down the tube from rest under the influence of gravity. Prove that in any position, the reaction of the tube is equal to  $2w \left(\frac{h+a}{\rho}\right)$ , where ' $w$ ' is the weight of the particle, ' $\rho$ ' the radius of curvature of the tube, ' $4a$ ' its latus rectum and ' $h$ ' the initial vertical height of the particle above the vertex of the tube. **[12 Marks]**
62. A straight uniform beam of length ' $2h$ ' rests in limiting equilibrium in contact with a rough vertical wall of height ' $h$ ' with one end on a rough horizontal plane and with the other end projecting beyond the wall. If both the wall and the plane be equally rough, prove that ' $\lambda$ ' the angle of friction, is given by  $\sin 2\lambda = \sin \alpha \sin 2\alpha$ , ' $\alpha$ ' being the inclination of the beam to the horizon. **[12 Marks]**
63. A particle  $P$  moves in a plane such that it is acted on by two constant velocities  $u$  and  $v$  respectively along the direction  $OX$  and along the direction perpendicular to  $OP$  where  $O$  is same fixed point, the origin. Show that the path traversed by  $P$  is a conic section with focus at  $O$  and eccentricity  $\frac{u}{v}$ . **[15 Marks]**
64. A particle of mass  $m$  moves under a force  $m\mu \left\{ 3au^4 - 2(a^2 - b^2)u^5 \right\}$ ,  $u = \frac{1}{r}$ ,  $a > b$ ,  $a, b$  and  $\mu (> 0)$  being given constants. It is projected from an apse at a distance  $a + b$  with velocity  $\frac{\sqrt{\mu}}{a + b}$ . Show that its orbit is given by the equation  $r = a + b \cos \theta$ , where  $(r, \theta)$  are the plane polar coordinates of a point. **[15 Marks]**
65. A shell lying in a strength smooth horizontal tube suddenly breaks into two portions of masses  $m_1$  and  $m_2$  if  $s$  be the distance between the two masses inside the tube after time  $t$ , show that the work done by the explosion can be written as equal to  $\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} \frac{s^2}{t^2}$ . **[15 Marks]**
66. A ladder of weight  $10 \text{ kg}$ . rests on a smooth horizontal ground leaning against a smooth vertical wall at an inclination  $\tan^{-1} 2$  with the horizontal and is prevented from slipping by a string attached at its lower end and to the junction of the floor and the wall. A body of weight  $30 \text{ kg}$  begins to ascend the ladder. If the string can bear a tension of  $10 \text{ kg. wt.}$ , how far along the ladder can the boy rise with safety? **[15 Marks]**

67. A solid right circle cone whose height is  $h$  and radius of whose base is  $r$ , is placed on an inclined plane and it is prevented from sliding. If the inclination  $\theta$  of the plane (to the horizontal) be gradually decreased find when the cone will topple over. For a cone whose semi- vertical angle is  $30^\circ$  determine the circular value of  $\theta$  which when exceeded, the cone will topple over. [15 Marks]

## 2007

68. A particle falls from rest under gravity in a medium whose resistance varies as the velocity of the particle. Find the distance fallen by the particle and its velocity at time  $t$ . [12 Marks]
69. A uniform string of length one-meter hangs over two smooth pegs  $P$  and  $Q$  at different heights. The parts which hang vertical are of length 34 cm and 26 cm. find the ratio in which the vertex of the Catenary divides the whole string. [12 Marks]
70. A quadrant of the ellipse  $x^2 + 4y^2 = 4$  is just immersed vertically in a homogeneous liquid with the major axis in the surface. Find the centre of pressure. [12 Marks]
71. A particle is performing simple harmonic motion of period  $T$  about a centre  $O$ . It passes through a point  $P(OP = p)$  with velocity  $v$  in the direction  $OP$ . Show that the time which elapses before it returns to  $P$  is  $\frac{T}{\pi} \tan^{-1} \left( \frac{uT}{2\pi p} \right)$ . [15 Marks]
72. A particle attached to a fixed peg  $O$  by a string of length  $l$  is lifted up with the string horizontal and then let go. Prove that when the string makes an angle  $\theta$  with the horizontal the resultant acceleration is  $g\sqrt{(1 + 3\sin^2 \theta)}$ . [15 Marks]
73. A uniform beam of length  $l$  rests with its ends on two smooth planes which intersect in a horizontal line if the inclination of the planes to the horizontal are  $\alpha$  and  $\beta$  ( $\beta > \alpha$ ) show that the inclination  $\theta$  of the beam to the horizontal, in one of the equilibrium positions, is given by  $\tan \theta = \frac{1}{2}(\cot \alpha - \cot \beta)$ , and show that the beam is unstable in this position. [15 Marks]
74. A cone whose vertical angle is  $\frac{\pi}{3}$  has its lowest generator horizontal and is filled with a liquid. Prove that the pressure on the curved surface  $\frac{W}{2}\sqrt{19}$  is where  $W$  the weight of the liquid is. [15 Marks]

## 2006

75. A particle is free to move on a smooth vertical circular wire of radius  $a$ . it is projected horizontal from the lowest point with velocity  $2\sqrt{ga}$ . Show that the reaction between the particle and the wire is zero after is time  $\sqrt{\frac{a}{g}} \log(\sqrt{5} + \sqrt{6})$ . [12 Marks]
76. The middle point of opposite sides of a jointed quadrilateral are connected by light rods of length  $l, l'$ . if  $T, T'$  be the tension in these rods, prove that  $\frac{T}{l} + \frac{T'}{l'} = 0$ . [12 Marks]
77. Find the depth of the centre of pressure of a triangular lamina with a vertex  $l$  in the surface of the liquid and other two vertices at depths  $b$  and  $c$  from the surface. [12 Marks]

78. A particle, whose mass is  $m$  is acted upon by a force  $m\left(x + \frac{a^4}{x^3}\right)$  towards the origin. If it starts from rest at a distance  $a$ , show that it will arrive at origin in time  $\frac{\pi}{4}$ . [15 Marks]
79. If  $u$  and  $v$  are the velocity of projection and the terminal velocity respectively of a particle rising vertically against a resistance varying as the square of the velocity, prove that the time taken by the particle to reach the highest point is  $\frac{v}{g} \tan^{-1}\left(\frac{u}{v}\right)$ . [15 Marks]
80. Show that the length of an endless chain which will hang over a circular pulley of radius  $c$  so as to be in contact with two-thirds of the circumference of the pulley is  $c\left\{\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right\}$ . [15 Marks]
81. A uniform rod of length  $2a$  can turn freely about one end which is fixed at a height  $h (< 2a)$  above the surface of the liquid. If the densities of the rod and liquid be  $\rho$  and  $\sigma$ , show that the rod can rest either in a vertical position or inclined at an angle  $\theta$  to the vertical such that  $\cos \theta = \frac{h}{2a} \sqrt{\frac{\sigma}{\rho - \sigma}}$ . [15 Marks]

## 2005

82. A body of mass  $(m_1 + m_2)$  moving in a straight line is split into two parts of masses  $m_1$  and  $m_2$  by an internal explosion which generates kinetic energy  $E$ . If after the explosion, the two parts move in the same line as before, find their relative velocity. [12 Marks]
83. If a number of concurrent forces be represented in magnitude and direction by the sides of a closed polygon, taken in order, then show that these forces are in equilibrium. [12 Marks]
84. A particle is projected along the inner side of a smooth vertical circle of radius  $a$  so that its velocity at the lowest point is  $u$ . Show that if  $2ag < u^2 < 5ag$  the particle will leave the circle before arriving at the highest point and will describe a parabola whose latus rectum is  $\frac{2(u^2 - 2ga)^3}{27g^3a^2}$ . [15 Marks]
85. Two particles connected by a fine string are constrained to move in a fine cycloid tube in a vertical plane. The axis of the cycloid is vertical with vertex upwards. Prove that tension in the string is constant throughout the motion. [15 Marks]
86. Two equal uniform rods  $AB$  and  $AC$  of length  $a$  each are freely joined at  $A$ , and are placed symmetrically over two smooth pegs on the same horizontal level at a distance  $c$  apart ( $3c < 2a$ ). A weight equal to the weight of a rod, is suspended from the joint  $A$  in the position of equilibrium, find the inclination of the either rod with the horizontal by the principle of virtual work. [15 Marks]
87. A rectangular lamina of length  $2a$  and breadth  $2b$  is completely immersed in a vertical plane, in a fluid so that its centre is at a depth  $h$  and the sides  $2a$  make an angle  $\alpha$  with the horizontal. Find the position of the centre of pressure. [15 Marks]

## 2004

88. A point moving with uniform acceleration describes distances  $s_1$  and  $s_2$  metres in successive intervals of time  $t_1$  and  $t_2$  seconds. Express the acceleration in terms of  $s_1, s_2, t_1$  and  $t_2$ . [12 Marks]
89. A non-uniform string hangs under gravity. Its cross-section at any point is inversely proportional to the tension at that point. Prove that the curve in which the string hangs is an arc of a parabola with its axis vertical. [12 Marks]

90. A circular area of radius  $a$  is immersed with its plane vertical and its centre at a depth  $c$ . Find position of its centre of pressure. [12 Marks]
91. Prove that the velocity required to project a particle from a height  $h$  to fall at a horizontal distance  $a$  from a point of projection is at least equal to  $\sqrt{g[\sqrt{a^2 + h^2} - h]}$ . [15 Marks]
92. A car of mass 750 kg is running up a hill of 1 in 30 at a steady speed of 36 km/hr; the friction is equal to the weight of 40 kg. Find the work done in 1 second. [15 Marks]
93. A uniform bar  $AB$  weights  $12N$  and rests with one part  $AC$  of length 8m, on a horizontal table and the remaining part  $CB$  projecting over the edge of the table if the bar is on the point of overbalancing when a weight of 5 N is placed on it at point 2m from  $A$  and a weight of 7 N is hung from  $B$  find the length of  $AB$ . [15 Marks]
94. A cone, of given weight and volume, floats with its vertex downwards. Prove that the surface of the cone in contact with the liquid is least when its vertical angle is  $2 \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ . [15 Marks]

## 2003

95. A sphere of weight  $W$  and radius  $a$  lies within a fixed spherical shell of radius  $b$ . A particle of weight  $w$  is fixed to the upper end of the vertical diameter. Prove that equilibrium is stable if  $\frac{W}{w} > \frac{b-2a}{a}$ . [12 Marks]
96. A particle describes the curve  $r = a(1 + \cosh \theta) / (\cosh \theta - 2)$  under a force  $F$  to the pole. Show that the law of force is  $F \propto \frac{1}{r^4}$ . [12 Marks]
97. An elastic string of natural length  $a + b$ , where  $a > b$  and modulus of elasticity  $\lambda$  has particle of mass  $m$  attached to it at a distance  $a$  from one end which is fixed to a point  $A$  of a smooth horizontal plane. The other end of the string is fixed to a point  $B$  so that the string is just unstretched. If the particle be held at  $B$  and then released find the periodic time and the distance in which the particle will oscillate to and fro. [15 Marks]
98. If a particle slides down a smooth cycloid, starting from a point whose actual distance from the vertex is  $b$ , prove that its speed at any time  $t$  is  $\frac{2 \times b}{T} \sin\left(\frac{2xt}{T}\right)$  where  $T$  is the time of complete oscillation of the particle. [15 Marks]
99. A ladder on a horizontal floor lean against a vertical wall the coefficients of friction of the floor and the wall with the ladder are  $\mu$  and  $\mu'$  respectively If a man, whose weight is  $n$  times that of the ladder, wants to climb up the ladder, find the minimum safe angle of the ladder with the floor. [15 Marks]
100. An ellipse is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  immersed vertically in a fluid with its semi-axis of length  $a$  horizontal. If its centre be at a depth  $h$ , find the depth of the centre of pressure. [15 Marks]

## 2002

101. A particle, whose mass is  $m$  is acted upon by a force  $m\left(x + \frac{a^4}{x^3}\right)$  towards the origin. If it starts from rest at a distance  $a$ , from the origin. Show that the time taken by it to reach the origin is  $\frac{\pi}{4}$ . [12 Marks]

102. Obtain the equation of the curve in which a string hangs under gravity from two fixed points (not lying in a vertical line) when line mass density at each of its points varies as the radius of curvature of the curve. [12 Marks]
103. Half of the ellipse is vertically immersed in water with minor axis just in the surface. Find the position of centre of pressure. [12 Marks]
104. A heavy particle of mass  $m$  slides on a smooth arc of a cycloid in a medium whose resistance is  $\nu$ , being the velocity of the particle and  $c$  being the distance of the starting point from the vertex upwards, find the velocity of the particle at the cusp. [15 Marks]
105. A particle describes a curve with constant velocity and its angular velocity about a given point  $O$  varies inversely as its distance from  $O$ . Show that the curve is an equiangular spiral. [15 Marks]
106. Five weightless rods of equal length are jointed together so as to form a rhombus  $ABCD$  with a diagonal  $BD$ . If a weight  $W$  be attached to  $C$  and the system be suspended from a point  $A$ , show that the thrust in  $BD$  is equal to  $\frac{W}{\sqrt{3}}$ . [15 Marks]
107. A solid cylinder floats in a liquid with its axis vertical. Let  $\sigma$  be the ratio of the specific gravity of the cylinder to that of the liquid. Prove that the equilibrium is stable if the ratio of the radius of the base to the height is greater than  $\sqrt{2\sigma(1-\sigma)}$ . [15 Marks]

## 2001

108. Find the law of force to the pole when the path of a particle is the cardioid  $r = a(1 - \cos \theta)$  and prove that if  $F$  be the force at the sphere and  $v$  the velocity there then  $3v^2 = 4aF$  [12 Marks]
109. The middle points of opposite sides of a jointed quadrilateral are connected by light rods of length  $l, l'$ . If  $T, T'$  be the tension in these rods, prove that  $\frac{T}{l} + \frac{T'}{l'} = 0$ . [12 Marks]
110. A solid right circular cone with semi-vertical angle  $\alpha$  is just immersed in a liquid with a generating line on the surface if  $\theta$  be the inclination of the vertical with the resultant thrust on the curve surface, prove that  $(1 - 3\sin^2 \alpha) \tan \theta = 3\sin \alpha \cos \alpha$ . [12 Marks]
111. A comet describing a parabola under inverse square law about the sun, when nearest to it suddenly breaks up, without gain or loss of kinetic energy, into two equal portions one of which describes a circle. Prove that the other will describe a hyperbola of eccentricity 2. [15 Marks]
112. A particle of mass  $M$  is at rest and begins to move under the action of a constant force  $F$  in a fixed direction. It encounters the resistance of a stream of fine dust moving in the opposite direction with velocity  $V$  which deposits matter on it at a constant rate  $\rho$ . Show that the mass of the particle will be  $m$  when it has travelled a distance  $\frac{k}{\rho^2} \left[ m - M \left\{ 1 + \log_e \frac{m}{M} \right\} \right]$  where  $K = F - \rho V$ . [15 Marks]
113.  $OA, OB$  and  $OC$  are edges of a cube of side  $a$ , and  $OO', AA', BB'$  and  $CC'$  are its diagonals. Along  $OB', O'A, BC$  and  $C'A$  act forces equal to  $P, 2P, 3P$  and  $4P$  respectively. Reduce the system to a force at  $O$  together with a couple. [15 Marks]
114. A right circular cylinder floating with its axis horizontal and in the surface is displaced in the vertical plane through the axis. Discuss its stability of equilibrium. [15 Marks]

## 2000

115. If  $T$  is the tension at any point  $P$  of a catenary and  $T_0$  that at the lowest point  $C$  show that  $T^2 - T_0^2 = W^2$ , where  $W$  is the weight of the arc  $CP$  of the catenary. [12 Marks]

116. Prove that a central force motion is a motion in a plane and the areal velocity of a particle is constant. [12 Marks]
117. A trapezoidal plate having its parallel sides of length  $x$  and  $y$ , ( $x > y$ ) at a distance  $z$  apart, is immersed vertically in water into  $x$  side uppermost (horizontal) at a depth  $d$  below the water surface. Find the total thrust on the surface. [12 Marks]
118. A telephone wire weighing 0.04 lb per foot has a horizontal span of 150 feet and sag of 1.5 feet. Find the length of the wire and also find maximum tension. [15 Marks]
119. Assuming that the earth attracts points inside it with a force which varies as the distance from its centre, show that if a straight frictionless airless tunnel be made from one point of the earth's surface to any point, a train would traverse the tunnel in slightly less than three quarter of an hour. Assume the earth to be a homogeneous sphere of radius 6400 km. [15 Marks]
120. A small bead is projected with any velocity along the smooth circle wire under the action of a force varying inversely as the fifth power of the distance from an centre of force situated on the circumference. Power that the pressure on the wire is constant. [15 Marks]
121. A conical buoy 1 meter long, and of base diameter 1.2 meter, floats in water with its apex downwards. Determine the minimum weight of the buoy, for stable equilibrium. [15 Marks]

## 1999

122. A perfectly rough plane is inclined at an angle  $\alpha$  to the horizon. Show that the least eccentricity of the ellipse which can rest on the plane is  $\left[ \frac{2\sin\alpha}{1+\sin\alpha} \right]^{\frac{1}{2}}$ . [20 Marks]
123. A string of length  $a$  forms the shorted diagonal of a rhombus formed of four uniform rods, each of length  $b$  and weight  $w$ , are hinged together. If one of the rods be supported in horizontal position, prove that the tension of the string is  $\frac{2w(2b^2 - a^2)}{b(4b^2 - a^2)^{\frac{1}{2}}}$ . [20 Marks]
124. A uniform chain, of length  $l$  and weight  $W$  hangs between two fixed points at the same level and weight  $W'$  is attached at the middle point. If  $K$  be the sag in the middle prove that pull on either point of support is  $\frac{K}{2l}W + \frac{l}{4K}W' + \frac{l}{8K}W$ . [20 Marks]
125. If in a simple harmonic motion  $u, v, w$  be the velocities at a distance  $a, b$  and  $c$  from a fixed point on the straight line (which is not the centre of force). Show that the period  $T$  is given by the equation  $\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ . [20 Marks]
126. A particle move with a centre acceleration  $\mu(r^5 - c^4r)$  being projected from an apse at a distance  $c$  with a velocity  $\sqrt{\frac{2\mu}{3}}c^3$ . Determine its path. [20 Marks]
127. A particle of mass  $m$  projected vertically under gravity, the resistance of air being  $mk$  (velocity). Show that the greatest height attained by the particle is  $V^2 / g[\lambda - \log(1 + \lambda)]$  where  $V$  is the terminal velocity of the particle and  $\lambda V$  is the initial vertical velocity.
128. An ellipse is just immersed in water (touching water surface) with its major axis vertical. Show that if the centre of pressure coincides with the focus the eccentricity of ellipse  $\frac{1}{4}$ . [20 Marks]

129. Two solids are each weighed in succession in three homogeneous liquid of different densities. If the weight of the one are  $w_1, w_2$  and  $w_3$ , and those of the other are  $W_1, W_2, W_3$ . Prove that
- $$w_1(W_2 - W_3) + w_2(W_3 - W_1) + w_3(W_1 - W_2) = 0. \quad [20 \text{ Marks}]$$

130. Masses  $m$  and of two gasses in which the ration of the pressure of the density  $\left(\frac{p}{\rho}\right)$  are respectively  $k$  and  $k'$ , are mixed at the same temperature. Prove that the ration of the pressure to the density ion the compound is  $\frac{mk + m'k'}{m + m'}$ . [20 Marks]

131. Drive the Lorentz transformation equations. [20 Marks]

132. If  $u$  and  $v$  are two velocities in the same direction and  $V$  is their resultant velocity given by  $\tanh^{-1} \frac{V}{c} = \tanh^{-1} \frac{u}{c} + \tanh^{-1} \frac{v}{c}$ , then deduce the law composition of velocities from this equation. [20 Marks]

133. Defined relativistic energy and momentum and establish  $E^2 = p^2c^2 + m_0^2c^4$  with usual notation. [20 Marks]

134. Two lumps of clay each of rest mass  $m_0$  collide head on with velocity  $\frac{3}{5c}$ , and stick together. What is the mass of the composite lump?

## 1998

135. A heavy elastic string whose natural length is  $2\pi a$  is placed round a smooth one whose axis is vertical and whose semi-vertical angle is  $\alpha$  if  $w$  be the weight and  $\lambda$  the modulus of elasticity of the string. Prove that it be in equilibrium when in the form of a circle whose radius is  $a\left(1 + \frac{w}{2\pi\lambda} \cot \alpha\right)$ . [20 Marks]

136. Show how to cut of uniform cylinder a cone whose base coincides with that of a cylinder, so that the centre of gravity of the remaining solid may coincide with the vertex of the cone. [20 Marks]

137. One end of an inextensible string is fixed to a point  $O$  and to the other end is tied a particle of mass  $m$  the particle is projected from its position of equilibrium vertically below  $O$  with a horizontal velocity so as to carry it right round the circle prove that the some of the tensions at the end of a diameter is constant. [20 Marks]

138. Two particles of masses  $m_1$  and  $m_2$  moving in coplanar parabolas round the sun collide at right coalesce when their common distance from the sun is  $R$  show that the subsequent path of the combine particles is an ellipse of major axis  $(m_1 + m_2)^2 \frac{R}{2m_1m_2}$ . [20 Marks]

139. A right circle cone of density  $\rho$  floats just immersed with its vertex downwards in a vessel containing two liquids of densities  $\sigma_1$  and  $\sigma_2$  respectively. Show that the plane of separation of the two liquids cut off from the axis of the cone a fraction  $\left[\frac{\rho - \sigma_2}{\sigma_1 - \sigma_2}\right]^{\frac{1}{3}}$  of its length. [20 Marks]

140. A cone floats with its axis horizontal in a liquid of density double its own. Fine the pressure on its base prove that the if  $\theta$  be the inclination to the vertical of the resultant thrust on the curved surface and  $\alpha$  the semi-vertical angle of the cone, then  $\theta = \tan^{-1} \left[\frac{4}{\pi} \tan \alpha\right]$ . [20 Marks]

# 1997

141. A heavy uniform chain rests on a rough cycloid whose axis vertical and vertex upwards, one end of the chain being at the vertex and the other at a cusp. If the equilibrium is limiting show that  $(1 + \mu^2)e^{\mu x/2}$ . [20 Marks]
142. A solid frustum of a paraboloid of revolution of height  $h$  and latus rectum  $4a$  rest with its vertex on the vertex of another paraboloid (inverted) of revolution whose latus rectum is  $4b$ . Show that equilibrium is state if  $h < \frac{3ab}{a+b}$ . [20 Marks]
143. A cylinder of wood (specific gravity  $\frac{3}{4}$ ) of height  $h$  floats with its axis vertical in water and oil (specific gravity  $\frac{1}{2}$ ) the length of the solid in contact with the oil is a  $\left(\frac{h}{2}\right)$  find how much of the wood is above the liquid Also find to what additional depth much oil be added so to cover the cylinder. [20 Marks]
144. A shell bursts on contact with the ground and pieces from it fly in all direction with all velocities up to 80 units show that a man 100 units away is danger for a time of  $\frac{5}{2}\sqrt{2}$  units if  $g$  is assumed to be of 32 units. [20 Marks]
145. A particle moves under a force  $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}$ ,  $a > b$  and is projected from an apse at a distance  $a + b$  with velocity  $\sqrt{\frac{\mu}{a+b}}$  Find the orbit. [20 Marks]
146. A particle is projected along the inner side of a smooth circle of radius  $a$  the velocity at the lowest point being  $u$  show that  $2ag < u^2 < 5ag$ , the particle leaves the circle before arriving at the highest point what is the nature of the path after the particle leaves the circle? [20 Marks]

# 1996

147. A body of weight  $W$  is placed on a rough inclined plane whose inclination of the horizon is  $\alpha$  greater than then angle of friction  $\lambda$ . The body is supported by a force acting in a vertical plane through the line of greatest slope and make an angle  $\theta$  with the inclined plane. Find the limits between which the force must lie. [20 Marks]
148. A body consisting of a cone and a hemisphere on the same base rests on a rough horizontal ... table. The hemisphere being in contact with the table Show that the greatest height of the cone, so that the equilibrium may be stable is  $\sqrt{3}$  times the radius of the sphere. [20 Marks]
149. A hollow cone without weight closed and filled with a liquid is suspended from a point in the rim of its base If  $\phi$  be the angle which the direction of the resultant pressure makes with the vertical then show that  $\cot \phi = \frac{28 \cot \alpha + \cot^3 \alpha}{48}$ ,  $\alpha$  being the semi vertical angle of the cone. [20 Marks]
150. One end of a light elastic string of natural length  $a$  and modulus  $2mg$  is attached to a fixed point  $O$  and the other to a particle of mass  $m$ . The particle is allowed to fall from the position of rest at  $O$ . Find the greatest extension of the sting and show that the particle will reach  $O$  again after a time  $(\pi + 2 - \tan^{-1} 2)\sqrt{\frac{2a}{g}}$  [20 Marks]
151. A stone is thrown at an angle  $\alpha$  with the horizon from a point in an inclined plane whose inclination to the horizon  $\beta$ , the trajectory lying in the vertical plane containing the line of greatest slopes. show that if  $\theta$  be

the elevation of that point  $bf$  the path which is most distance from the inclined plane then

$$2 \tan \theta = \tan \alpha + \tan \beta.$$

[20 Marks]

152. A particle moves under gravity on a vertical circle, sliding down the convex side of smooth circular arc if its initial velocity is that due to a fall to the starting point from a height  $h$  above the center, show that it will fly off the circle when at a height  $\frac{2h}{3}$  above the center. [20 Marks]

## 1995

153. Prove that for the common Catenary the radius of curvature at any point of the curve is equal to the length of the normal intercepted between the curve and the directrix. [20 Marks]
154. Two uniform rods  $AB$  and  $AC$  smoothly jointed at  $A$  are in equilibrium in a vertical plane the ends  $B$  and  $C$  rest on a smooth horizontal plane and the middle points of  $AB$  and  $AC$  are connected by a string show that the tension of the string is  $\frac{W}{(\tan B + \tan C)}$  where  $W$  is the total weight of the rods and  $B$  and  $C$  are the inclination to the horizontal of the rods  $AB$  and  $AC$  [20 Marks]
155. A semi-ellipse bounded by its minor axis is just immersed in a liquid the density of which varies as the depth. If the minor axis be in the surface find the eccentricity in order that the focus may be the centre of pressure. [20 Marks]
156. Two bodies, of masses  $M$  and  $M'$  are attached to the lower end of an elastic string whose upper end is fixed and hang at rest  $M$ , falls off. Show that the distance of  $M$  from the upper end of the string at time  $t$  is  $a + b + c \cos\left(\sqrt{\frac{g}{b}}t\right)$ , where  $a$  is the unstretched length of the string, and  $b$  and  $c$  the distances by which it would be stretched when supporting  $M$  and  $M'$  respectively. [20 Marks]
157. A particle of mass  $m$  moved under a centre attractive force  $m\mu\left(\frac{5}{r^3} + \frac{8c^2}{r^5}\right)$  and is projected from an apse at a distance with velocity  $3\sqrt{\frac{\mu}{c}}$  prove that the orbit is  $r = c \cos\left(\frac{2\theta}{3}\right)$  and that it will arrive at the origin after a time  $\frac{\pi c^2}{8\sqrt{\mu}}$ . [20 Marks]
158. If  $t$  be the time which a projectile reaches a point  $P$  in its path and  $t'$  the time from  $P$  till it reaches the horizontal plane through the point of projection show that the height of  $P$  above the horizontal plane is  $\frac{1}{2}gt^2$  and the maximum height is  $\frac{1}{8}g(t+t')^2$ . [20 Marks]

## 1994

159. Show that the length of an endless chain which will hang over a circular pulley of radius  $a$  so as to be in contact with two third of the circumference of the pulley is  $a\left\{\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right\}$  [20 Marks]
160. A smooth rod passes through a smooth ring at the focus ellipse whose major axis is horizontal and rests with its lower end on the quadrant of the curve which is farthest removed from the focus Find its position of equilibrium and show that its length must at least be  $\left(\frac{3a}{4} + \frac{a}{4}\sqrt{1+8e^2}\right)$  where  $2a$  is the major axis and  $e$  be the eccentricity. [20 Marks]

161. The height of a balloon is calculated from the barometric pressure reading ( $p$ ) on the assumption that the pressure of the air varies  $n^{th}$  as the density. Show that if the pressure actually varies as then power of the density there will be an error  $h_0 \left[ \frac{n}{n-1} \left\{ 1 - \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} \right\} - \log \frac{p}{p_0} \right]$  in the calculated height where  $h_0$  is the height of the homogeneous atmosphere and  $p_0$  is the pressure at the surface of the earth. [20 Marks]
162. If in a simple harmonic motion, the velocities at distances point  $a, b, c$  from a fixed point on the straight line which is not the center of force be  $u, v, w$  respectively show that the periodic time  $T$  is given by  $\frac{4\pi^2}{T^2} (b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ . [20 Marks]
163. A gun is firing from the sea-level out to sea it is mounted in a battery  $h$  meters high up and fired at the same elevation  $\alpha$ . Show that the range is increased by  $\frac{1}{2} \left[ \left( 1 + \frac{2gh}{u^2 \sin^2 \alpha} \right)^{1/2} - 1 \right]$  of itself  $u$  being the velocity of projectile. [20 Marks]
164. A particle of mass  $m$  is projected vertical under gravity the resistance of the air being  $mk$  time the velocity, show that the greatest height attained by the particle  $\frac{V^2}{g} [\lambda - \log(1 + \lambda)]$  is where  $V$  is the terminal velocity of the particle and  $\lambda V$  is the initial vertical velocity. [20 Marks]
- 1993**
165. The end links of a uniform chain slide along a fixed rough horizontal rod prove that the ratio of the maximum span to the length of the chain is  $\mu \log \left( \frac{1 + \sqrt{1 + \mu^2}}{\mu} \right)$ , where  $\mu$  is the coefficient of friction. [20 Marks]
166. A solid hemisphere is supported by a string to a fixed point on its rim and to point on a smooth vertical wall with which the curved surface of the sphere is in contact if  $\theta$  and  $\phi$  are the inclination of the string and the plane base of the hemisphere to the vertical, prove that  $\tan \phi = \frac{3}{8} + \tan \theta$ . [20 Marks]
167. A semicircular lamina is completely immersed in water with its plane vertical, so that the extremity  $A$  of its bounded diameter is in the surface and the diameter make with the surface and angle  $\alpha$  prove that if  $E$  be the center of pressure and  $\phi$  the angle between  $AE$  and the diameter  $\tan \phi = \frac{3\pi + 16 \tan \alpha}{16\pi + 15\pi \tan \alpha}$ . [20 Marks]
168. A point executes simple harmonic motion such that in two of its positions the velocities are  $u$  and  $v$  and the corresponding accelerations are  $\alpha$  and  $\beta$  show that the distance between the position is  $\frac{v^2 - u^2}{\alpha + \beta}$ . [20 Marks]
169. A particle moves under a force  $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}$ ,  $a > b$  and is projected from an apse at a distance  $a + b$  with a velocity  $\sqrt{\frac{\mu}{a+b}}$ . Show that its orbit is  $r = a + b \cos \theta$ . [20 Marks]

170. A particle is projected upward with a velocity  $u$  in a medium whose resistance varies as the square of the velocity. Prove that it will return to the point of projection with velocity  $v = \frac{uV}{\sqrt{u^2 + V^2}}$  after a time

$$\frac{V}{g} \left( \tan^{-1} \frac{u}{V} \tanh^{-1} \frac{u}{V} \right), \text{ where } V \text{ is the terminal velocity.}$$

[20 Marks]

## 1992

171. Two equal rods, each of weight  $wl$  and length  $l$  are hinged together and placed astride smooth horizontal cylindrical peg of radius  $r$ , Then the lower ends are tied together by a string and the rods are left at the same inclination  $\phi$  to the horizontal. Find the tension in the string and if the string is slack show that  $\phi$  satisfies the equation  $\tan^3 \phi + \tan \phi = \frac{1}{2r}$ . [20 Marks]
172. Defined central axis for a system of forces acting on a rigid body  $A$  force  $F$  acts along the axis of  $x$  and another force  $nF$  along a generator of the cylinder  $x^2 + y^2 = a^2$  show that the central axis lies on the cylinder.  $n^2(nx - z)^2 + (1 + n^2)^2 y^2 x = n^4 a^2$ . [20 Marks]
173. A semicircular of radius  $a$  is immersed vertically with its diameter horizontal at a depth  $b$ . if the circumference below the center, prove that the depth of center of pressure is  $\frac{3\pi(a^2 + 4b^2) + 32ab}{4(3b\pi + 4)}$  [20 Marks]
174. A particle is moving with center acceleration  $\mu(r^5 - c^4 r)$  being projected from an apse at a distance  $c$  with a velocity  $\sqrt{\left(\frac{2\mu}{3}\right)} c^3$ . Show that its path is the curve  $x^4 + y^4 = c^4$ . [20 Marks]
175. A particle is projected with a velocity whose horizontal and vertical components are respectively  $u$  and  $v$  from a given point in a medium whose resistance per unit mass is  $K$  times the speed obtain the equation of the path and prove that if  $K$  is small the horizontal range is approximately  $\frac{2uv}{g} - \frac{8uv^2 k}{3g}$ . [20 Marks]
176. A particle slides down the arc of a smooth vertical circle of radius  $a$  being slightly displaced from rest at the highest point of the circle. Find the point where it will strike the horizontal plane through the lowest point of the circle. [20 Marks]