Ordinary Differential Equations (P.D.E)

Previous year Questions from 2020 to 1992

			2021-22

2020

1. Solve the following differential equation: $x\cos\left(\dfrac{y}{x}\right)(ydx + xdy) = y\sin\left(\dfrac{y}{x}\right)(xdy - ydx)$ **[10 Marks]**

2. Find the orthogonal trajectories of the family of circles passing through the points $(0,2)$ and $(0,-2)$ **[10 Marks]**

3. Using the method of variation of parameters, solve the differential equation $y'' + (1 - \cot x)y' - y\cot x = \sin^2 x$ if $y = e^{-x}$ is one solution of CF. **[20 Marks]**

4. Using Laplace transform, solve the initial value problem $ty'' + 2ty' + 2y = 2$; and $y(0) = 1$ and $y'(0)$ is arbitrary. Does this problem have a unique solution? **[10 Marks]**

5. Solve the following differential equation: $(x+1)^2 y'' - 4(x+1)y' + 6y = 6(x+1)^2 + \sin\log(x+1)$ **[10 Marks]**

6. Find the general and singular solutions of the differential equation $9p^2(2+y)^2 = 4(3-y)$ where $p = \dfrac{dy}{dx}$ **[10 Marks]**

2019

7. Solve the differential equation $(2y\sin x + 3y^4 \sin x\cos x)dx - (4y^3\cos^2 x + \cos x)dy = 0$ **[10 Marks]**

8. Determine the complete solution of the differential $\dfrac{d^2 y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 3x^2 e^{2x}\sin 2x$ **[10 Marks]**

9. Solve the differential equation $\dfrac{d^2 y}{dx^2} + (3\sin x - \cot x)\dfrac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$ **[10 Marks]**

10. Find the Laplace transforms of $t^{-1/2}$ and $t^{-1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n \in N$ is

$$\dfrac{\Gamma\left(n+1+\dfrac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$$ **[10 Marks]**

11. Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2 y'' - 2xy' + 2y = x^3 \sin x$ and then find the general solution of the given equation by the method of variation of parameters. **[15 Marks]**

12. Obtain the singular solution of the differential equation

$$\left(\dfrac{dy}{dx}\right)^2\left(\dfrac{y}{x}\right)^2\cot^2\alpha - 2\left(\dfrac{dy}{dx}\right)\left(\dfrac{y}{x}\right) + \left(\dfrac{y}{x}\right)^2\cos ec^2\alpha = 1.$$ Also find the complete primitive of the given

differential equation. Give the geometrical interpretations of the complete primitive and singular solutions **[15 Marks]**

2018

13. Solve: $y'' - y = x^2 e^{2x}$ **[10 Marks]**

14. Solve: $y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$ **[10 Marks]**

15. (i) Find the Laplace transform of $f(t) = \dfrac{1}{\sqrt{t}}$. **[10 Marks]**

    (ii) Find the Inverse Laplace transform of $\dfrac{5s^2 + 3s - 16}{(s-1)(s-2)(s+3)}$

16. Solve: $\left(\dfrac{dy}{dx}\right)^2 y + 2\dfrac{dy}{dx}x - y = 0$ **[13 Marks]**

17. Solve: $y'' + 16y = 32\sec 2x$ **[13 Marks]**

18. Solve: $(1+x)^2 y'' + (1+x)y' + y = 4\cos(\log(1+x))$ **[13 Marks]**

19. Solve the initial value problem

$y'' - 5y' + 4y = e^{2t}; y(0) = \dfrac{19}{12}, y'(0) = \dfrac{8}{3}$ **[13 Marks]**

20. Find α and β such that $x^\alpha y^\beta$ an integrating factor of $(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ and solve the equation. **[12 Marks]**

21. Find $f(y)$ such that $(2xe^y + 3y^2)dy + (3x^2 + f(y))dx = 0$ is exact and hence solve. **[12 Marks]**

2017

22. Find the differential equation representing the entire circle in the xy-plane. **[10 Marks]**

23. Solve the following simultaneous liner differential equations: $(D+1)y = z + e^x$ and $(D+1)z = y + e^x$

where y and z are functions of independent variable x and $D \equiv \dfrac{d}{dx}$. **[8 Marks]**

24. If the growth rate of the population of bacteria at time t is proportional to the amount present at the time and population doubles in one week, then how much bacteria's can be expected after 4 weeks? **[8 Marks]**

25. Consider the differential equation $xyp^2 - (x^2 + y^2 - 1)p + xy = 0$ where $p = \dfrac{dy}{dx}$ substituting $u = x^2$ and

$v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p' = \dfrac{dv}{du}$ hence or otherwise solve

the equation. **[10 Marks]**

26. Solve the following initial value differential equations $20y'' + 4y' + y = 0, y(0) = 3.2, y'(0) = 0$. **[7 Marks]**

27. Solve the differential equation: $x\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 4x^3 y = 8x^3 \sin(x^2)$ **[9 Marks]**

28. Solve that following differential equation using method of variation of parameters

$\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 44 - 76x - 48x^2$. **[8 Marks]**

29. Solve the following initial value problem using Laplace transform: $\dfrac{d^2y}{dx^2} + 9y = r(x), y(0) = 0, y'(0) = 4$

where $r(x) = \begin{cases} 8\sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x \geq \pi \end{cases}$. **[17 Marks]**

2016

30. Find a particular integral of $\dfrac{d^2y}{dx^2} + y = e^{x/2}\sin\dfrac{x\sqrt{3}}{2}$ **[10 marks]**

31. Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. **[10 marks]**

32. Solve $\{y(1 - x\tan x) + x^2 \cos x\}dx - xdy = 0$ **[10 marks]**

33. Using the method of variation of parameter solve the differential equation

$(D^2 + 2D + 1)y = e^{-x}\log(x), \quad \left[D \equiv \dfrac{d}{dx}\right]$ **[15 marks]**

34. Find the general solution of the equation $x^2 \dfrac{d^3y}{dx^3} - 4x \dfrac{d^2y}{dx^2} + 6 \dfrac{dy}{dx} = 4$ **[15 marks]**

35. Using Laplace transformation solves the following: $y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6$ **[10 marks]**

2015

36. Solve the differential equation: $x \cos x \dfrac{dy}{dx} + y(x \sin x + \cos x) = 1$ **[10 Marks]**

37. Solve the differential equation: $(2xy^4 e^y + 2xy^3 + y)dx + (x^2 y^4 e^y - x^2 y^2 - 3x)dy = 0$ **[10 Marks]**

38. Find the constant a so that $(x+y)^a$ is the integrating factor of
   $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential equation **[12 Marks]**

39. (i) Obtain Laplace Inverse transform of $\left\{ \ln\left(1 + \dfrac{1}{s^2}\right) + \dfrac{s}{s^2 + 25} e^{-5s} \right\}$

   (ii) Using Laplace transform, solve $y'' + y = t, y(0) = 1, y'(0) = -2$ **[6+6=12 Marks]**

40. Solve the differential equation $x = py - p^2$ where $p = \dfrac{dy}{dx}$ **[13 Marks]**

41. Solve $x^4 \dfrac{d^4y}{dx^4} + 6x^3 \dfrac{d^3y}{dx^3} + 4x^2 \dfrac{d^2y}{dx^2} - 2x \dfrac{dy}{dx} - 4y = x^2 + 2\cos(\log_e x)$ **[13 Marks]**

2014

42. Justify that a differential equation of the form: $\left[y + xf(x^2 + y^2) \right] dx + \left[yf(x^2 + y^2) - x \right] dy = 0$ where
   $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\dfrac{1}{x^2 + y^2}$ is
   an integrating factor for it. Hence solve this differential equation for $f(x^2 + y^2) = (x^2 + y^2)^2$ **[10 Marks]**

43. Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of
   tangency. **[10 Marks]**

44. Solve by the method of variation of parameters: $\dfrac{dy}{dx} - 5y = \sin x$ **(10 Marks]**

45. Solve the differential equation: $x^3 \dfrac{d^3y}{dx^3} + 3x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + 8y = 65 \cos(\log_e x)$ **[20 Marks]**

46. Solve the following differential equation: $x \dfrac{d^2y}{dx^2} - 2(x+1) \dfrac{dy}{dx} + (x+2)y = (x-2)e^{2x}$, when e^x is a
   solution to its corresponding homogeneous differential equation. **[15 Marks]**

47. Find the sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$, to have an
   integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence
   find the integrating factor for the differential equation of $(x^2 + xy)dx + (y^2 + xy)dy = 0$ and solve it. **[15 Marks]**

48. Solve the initial value problem $\dfrac{d^2y}{dt^2} + y = 8e^{-2t} \sin t, \ y(0) = 0, y'(0) = 0$ by using Laplace transform.

   [20 Marks]

2013

49. If y is a function of x, such that the differential coefficient $\dfrac{dy}{dx}$ is equal to $\cos(x+y)+\sin(x+y)$. Find out a relation between x and y, which is free from any derivative / differential. **[10 Marks]**

50. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a\sin n\theta,\ (r,\theta)$ being the plane polar coordinates. **[10 Marks]**

51. Solve the differential equation $(5x^3 + 12x^2 + 6y^2)dx + 6xy\,dy = 0$ **[15 Marks]**

52. Using the method of variation of parameters, solve the differential equation $\dfrac{d^2y}{dx^2} + a^2 y = \sec ax$ **[15 Marks]**

53. Find the general solution of the equation $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = \ln x \sin(\ln x)$ **[15 Marks]**

54. By using Laplace transform method, solve the differential equation $(D^2 + n^2)x = a\sin(nt+\alpha)$, $D^2 = \dfrac{d^2}{dt^2}$ subject to the initial conditions $x = 0$ and $\dfrac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. **[15 Marks]**

2012

55. Solve $\dfrac{dy}{dx} = \dfrac{2xye^{(x/y)^2}}{y^2(1+e^{(x/y)^2}) + 2x^2 e^{(x/y)^2}}$ **[12 Marks]**

56. Find the orthogonal trajectory of the family of curves $x^2 + y^2 = ax$ **[12 Marks]**

57. Using Laplace transforms, solve the initial value problem $y'' + 2y' + y = e^{-t},\ y(0) = -1, y'(0) = 1$ **[12 Marks]**

58. Show that the differential equation $(2xy\log y)dx + (x^2 + y^2\sqrt{y^2+1})dy = 0$ is not exact. Find an integrating factor and hence, the solution of the equation **[20 Marks]**

59. Find the general solution of the equation $y''' - y'' = 12x^2 + 6x$ **[20 Marks]**

60. Solve the ordinary differential equation $x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$ **[20 Marks]**

2011

61. Obtain the solution of the ordinary differential equation $\dfrac{dy}{dx} = (4x+y+1)^2$, if $y(0) = 1$ **[10 Marks]**

62. Determine the orthogonal trajectory of a family of curves represented by the polar equation $r = a(1-\cos\theta),\ (r,\theta)$ being the plane polar coordinates of any point. **[10 Marks]**

63. Obtain Clairaut's form of the differential equation $\left(x\dfrac{dy}{dx} - y\right)\left(y\dfrac{dy}{dx} + x\right) = a^2\dfrac{dy}{dx}$. Also find its general solution **[15 Marks]**

64. Obtain the general solution of the second order ordinary differential equation $y'' - 2y' + 2y = x + e^x\cos x$, where dashes denote derivatives w.r.t. x **[15 Marks]**

65. Using the method of variation of parameters, solve the second order differential equation

$$\frac{d^2 y}{dx^2} + 4y = \tan 2x$$ **[15 Marks]**

66. Use Laplace transform method to solve the following initial value problem:

$$\frac{d^2 x}{dt^2} - 2\frac{dx}{dt} + x = e^t, \; x(0) = 2 \text{ and } \left.\frac{dy}{dt}\right|_{t=0} = -1$$ **[15 Marks]**

2010

67. Consider the differential equation $y' = \alpha x, \; x > 0$ where α is a constant. Show that
    (i) If $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;
    (ii) If $\alpha < 0$, then every solution tends to zero as $x \to \infty$ **[12 Marks]**

68. Show that the differential equation $(3y^2 - x) + 2y(y^2 - 3)y' = 0$ admits an integrating factor which is a function of $(x + y^2)$. Hence solve the equation **[12 Marks]**

69. Verify that $\frac{1}{2}(Mx + Ny)d\left[\log_e(xy)\right] + \frac{1}{2}(Mx - Ny)d\left[\log_e(x / y)\right] = Mdx + Ndy$. Hence show that-
    (i) If the differential equation $Mdx + Ndy = 0$ is homogeneous, then $(Mx + Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$;
    (ii) If the differential equation $Mdx + Ndy = 0$ is not exact but is of the form
    $f_1(xy)ydx + f_2(xy)xdy = 0$ then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx + Ny \equiv 0$;
    [20 Marks]

70. Use the method of undermined coefficients to find the particular solutions of
    $y'' + y = \sin x + (1 + x^2)e^x$ and hence find its general solution. **[20 Marks]**

2009

71. Find the Wronskian of the set of functions: $\left\{3x^3, \; \left|3x^3\right|\right\}$ on the interval $[-1, \; 1]$ and determine whether the set is linearly dependent on $[-1, \; 1]$ **[12 Marks]**

72. Find the differential equation of the family of circles in the xy- plane passing through $(-1, \; 1)$ and $(1, \; 1)$ **[20 Marks]**

73. Find the inverse Laplace transform of $F(s) = 1n\left(\frac{s+1}{s+s}\right)$ **[20 Marks]**

74. Solve: $\frac{dy}{dx} = \frac{y^2(x - y)}{3xy^2 - x^2 y - 4y^3}$, $y(0) = 1$ **[20 Marks]**

2008

75. Solve the differential equation $ydx + (x + x^3 y^2)dy = 0$ **[12 Marks]**

76. Use the method of variation of parameters to find the general solution of
    $x^2 y'' - 4xy' + 6y = -x^4 \sin x$ **[12 Marks]**

77. Using Laplace transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$, $y(0) = 1$, $y'(0) = -1$ **[15 Marks]**

78. Solve the differential equation $x^3 y'' - 3x^2 y' + xy = \sin(\ln x) + 1$ **[15 Marks]**

79. Solve the equation $y - 2xp + yp^2 = 0$, where $p = \dfrac{dy}{dx}$ **[15 Marks]**

2007

80. Solve the ordinary differential equation $\cos 3x \dfrac{dy}{dx} - 3y \sin 3x = \dfrac{1}{2}\sin 6x + \sin^2 3x, 0 < x < \dfrac{\pi}{2}$ **[12 Marks]**

81. Find the solution of the equation $\dfrac{dy}{y} + xy^2 dx = -4x dx$ **[12 Marks]**

82. Determine the general and singular solutions of the equation $y = x\dfrac{dy}{dx} + a\dfrac{dy}{dx}\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}$, a being a constant. **[15 Marks]**

83. Obtain the general solution of $[D^3 - 6D^2 + 12D - 8]y = 12\left(e^{2x} + \dfrac{9}{4}e^{-x}\right)$, where $D \equiv \dfrac{dy}{dx}$ **[15 Marks]**

84. Solve the equation $2x^2 \dfrac{d^2 y}{dx^2} + 3x\dfrac{dy}{dx} - 3y = x^3$ **[15 Marks]**

85. Use the method of variation of parameters to find the general solution of the equation $\dfrac{d^2 y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 2e^x$ **[15 Marks]**

2006

86. Find the family of curves whose tangents form an angle $\dfrac{\pi}{4}$ with the hyperbolas $xy = c, c > 0$ **[12 Marks]**

87. Solve the differential equation $\left(xy^2 + e^{-\frac{1}{x^3}}\right)dx - x^2 y dy = 0$ **[12 Marks]**

88. Solve: $(1 + y^2) + (x - e^{-\tan^{-1} y})\dfrac{dy}{dx} = 0$ **[15 Marks]**

89. Solve the equation $x^2 p^2 + py(2x + y) + y^2 = 0$ using the substitution $y = u$ and $xy = v$ and find its singular solution, where $p = \dfrac{dy}{dx}$ **[15 Marks]**

90. Solve the differential equation $x^2 \dfrac{d^3 y}{dx^3} + 2x\dfrac{d^2 y}{dx^2} + 2\dfrac{y}{x} = 10\left(1 + \dfrac{1}{x^2}\right)$ **[15 Marks]**

91. Solve the differential equation $(D^2 - 2D + 2)y = e^x \tan x$, $D \equiv \dfrac{dy}{dx}$ by the method of variation of parameters. **[15 Marks]**

2005

92. Find the orthogonal trajectory of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter. **[12 Marks]**

93. Solve: $xy\dfrac{dy}{dx} = \sqrt{(x^2 - y^2 - x^2y^2 - 1)}$ **[12 Marks]**

94. Solve the differential equation: $\left[(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)\right]y = \dfrac{1}{(x+1)}$ **[15 Marks]**

95. Solve the differential equation: $(x^2 + y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$ where $p = \dfrac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. **[15 Marks]**

96. Solve the differential equation $(\sin x - x\cos x)y'' - x\sin x\,y' + y\sin x = 0$ given that $y = \sin x$ is a solution of this equation. **[15 Marks]**

97. Solve the differential equation $x^2y'' - 2xy' + 2y = x\log x,\ x > 0$ by variation of parameters **[15 Marks]**

2004

98. Find the solution of the following differential equation $\dfrac{dy}{dx} + y\cos x = \dfrac{1}{2}\sin 2x$ **[12 Marks]**

99. Solve: $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$ **[12 Marks]**

100. Solve: $(D^4 - 4D^2 - 5)y = e^x(x + \cos x)$ **[15 Marks]**

101. Reduce the equation $(px - y)(py + x) = 2p$, where $p = \dfrac{dy}{dx}$ to Clairaut's equation and hence solve it. **[15 Marks]**

102. Solve: $(x+2)\dfrac{d^2y}{dx^2} - (2x+5)\dfrac{dy}{dx} + 2y = (x+1)e^x$ **[15 Marks]**

103. Solve the following differential equation: $(1-x^2)\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} - (1+x^2)y = x$ **[15 Marks]**

2003

104. Show that the orthogonal trajectory of a system of con-focal ellipses is self-orthogonal **[12 Marks]**

105. Solve: $x\dfrac{dy}{dx} + y\log y = xye^x$ **[12 Marks]**

106. Solve $(D^5 - D) = 4(e^x + \cos x + x^3)$, where $D \equiv \dfrac{dy}{dx}$ **[15 Marks]**

107. Solve the differential equation $(px^2 + y^2)(px + y) = (P+1)^2$, where $p = \dfrac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions **[15 Marks]**

108. Solve $(1-x^2)y'' + (1+x)y' + y = \sin 2\left[\log(1+x)\right]$ **[15 Marks]**

109. Solve the differential equation $x^2y'' - 4x\,y' + 6y = x^4\sec^2 x$ by variation of parameters. **[15 Marks]**

2002

110. Solve: $x\dfrac{dy}{dx} + 3y = x^3 y^2$ **[12 Marks]**

111. Find the values of λ for which all solutions of $x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} - \lambda y = 0$ tend to zero as $x \to \infty$.

   [12 Marks]

112. Find the value of constant λ such that the following differential equation becomes exact.

   $(2xe^y + 3y^2)\dfrac{dy}{dx} + (3x^2 + \lambda e^y) = 0$. Further, for this value of λ, solve the equation. **[15 Marks]**

113. Solve: $\dfrac{dy}{dx} = \dfrac{x+y+4}{x-y-6}$ **[15 Marks]**

114. Using the method of variation of parameters, find the solutions of $\dfrac{d^2y}{d^2x} - 2\dfrac{dy}{dx} + y = xe^x \sin x$ with

   $y(0) = 0$ and $\left(\dfrac{dy}{dx}\right)_{x=0} = 0$ **[15 Marks]**

115. Solve: $(D-1)(D^2 - 2D + 2)y = e^x$ where $D \equiv \dfrac{dy}{dx}$ **[15 Marks]**

2001

116. A continuous function $y(t)$ satisfies the differential equation $\dfrac{dy}{dx} = \begin{cases} 1 + e^{1-t}, & 0 \le t < 1 \\ 2 + 2t - 3t^2, & 1 \le t < 5 \end{cases}$ If

   $y(0) = -e$ find $y(2)$ **[12 Marks]**

117. Solve: $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 3y = x^2 \log_e x$ **[12 Marks]**

118. Solve: $\dfrac{dy}{dx} + \dfrac{y}{x}\log_e y = \dfrac{y(\log_e y)^2}{x^2}$ **[15 Marks]**

119. Find the general solution of $ayp^2 + (2x - b)p - y = 0,\ a > 0$ **[15 Marks]**

120. Solve: $(D^2 + 1)^2 y = 24x\cos x$ given that $y = Dy = D^2 y = 0$ and $D^3 y = 12$ when $x = 0$ **[15 Marks]**

121. Using the method of variation of parameters, solve $\dfrac{d^2y}{dx^2} + 4y = 4\tan 2x$ **[15 Marks]**

2000

122. Show that $3\dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} - 8y = 0$ has an integral which is a polynomial in x. Deduce the general

   solution. **[12 Marks]**

123. Reduce $\dfrac{d^2y}{dx^2} + P\dfrac{dy}{dx} + Qy = R$, where P, Q, R are functions of x, to the normal form. Hence solve

   $\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$ **[15 Marks]**

124. Solve the differential equation $y = x - 2ap + ap^2$. Find the singular solution and interpret it geometrically **[15 Marks]**

125. Show that $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents a family of hyperbolas with a common axis and tangent at the vertex **[15 Marks]**

126. Solve $x\dfrac{dy}{dx} - y = (x-1)\left(\dfrac{d^2y}{dx^2} - x + 1\right)$ by the method of parameters **[15 Marks]**

1999

127. Solve the differential equation $\dfrac{xdx + ydy}{xdy - ydx} = \left(\dfrac{1 - x^2 - y^2}{x^2 + y^2}\right)^{1/2}$ **[20 Marks]**

128. Solve $\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} - 2y = e^x + \cos x$ **[20 Marks]**

129. By the method of variation of parameters solve the differential equation $\dfrac{d^2y}{dx^2} + a^2 y = \sec(ax)$ **[20 Marks]**

1998

130. Solve the differential equation: $xy - \left(\dfrac{dy}{dx}\right) = y^3 e^{-x^2}$ **[20 Marks]**

131. Show that the equation: $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents a family of hyperbolas having as asymptotes the lines $x + y = 0, \ 2x + y + 1 = 0$. **[20 Marks]**

132. Solve the differential equation: $y = 3px + 4p^2$ **[20 Marks]**

133. Solve the differential equation: $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = e^{4x}(x^2 + 9)$ **[20 Marks]**

134. Solve the differential equation: $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = x \sin x$ **[20 Marks]**

135. Solve the differential equation: $x^3\dfrac{d^3y}{dx^3} + 2x^2\dfrac{d^2y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right)$ **[20 Marks]**

1997

136. Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{x}{x^2 y + y^3}, \ y(0) = 0$ **[20 Marks]**

137. Solve $(x^2 - y^2 + 3x - y)dx + (x^2 - y^2 + x - 3y)dy = 0$ **[20 Marks]**

138. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. If its radius originally is 3mm, and one hour later has been reduced to 2 mm. find an expression for the radius of the rain drop at any time. **[20 Marks]**

139. Solve $\dfrac{d^4y}{dx^4} + 6\dfrac{d^3y}{dx^3} + 11\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} = 20e^{-2x}\sin x$ **[20 Marks]**

140. Make use of the transformation $y(x) = u(x)\sec x$ to obtain the solution of $y'' - 2y'\tan x + 5y = 0$, $y(0) = 0$, $y'(0) = \sqrt{6}$ **[20 Marks]**

141. Solve $(1 + 2x)^2 \dfrac{d^2 y}{dx^2} - 6(1 + 2x)\dfrac{dy}{dx} + 16y = 8(1 + 2x)^2$, $y(0) = 0$, $y'(0) = 2$ **[20 Marks]**

1996

142. Find the curves for which the sum of the reciprocals of the radius vector and polar sub tangent is constant. **[20 Marks]**

143. Solve: $x^2(y - px) = yp^2$, $p \equiv \dfrac{dy}{dx}$ **[20 Marks]**

144. Solve: $y\sin 2x\,dx - \left(1 + y^2 + \cos^2 x\right)dy = 0$ **[20 Marks]**

145. $\dfrac{d^2 y}{dx^2} + 2\dfrac{dy}{dx} + 10y + 37\sin 3x = 0$. Find the value of y when $x = \dfrac{\pi}{2}$, if it is given that $y = 3$ and $\dfrac{dy}{dx} = 0$ when $x = 0$ **[20 Marks]**

146. Solve: $\dfrac{d^4 y}{dx^4} + 2\dfrac{d^3 y}{dx^3} - 3\dfrac{d^2 y}{dx^2} = x^2 + 3e^{2x} + 4\sin x$ **[20 Marks]**

147. Solve: $x^3 \dfrac{d^3 y}{dx^3} + 3x^2 \dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} + y = x + \log x$ **[20 Marks]**

1995

148. Determine a family of curves for which the ratio of the y-intercept of the tangent to the radius vector is a constant. **[20 Marks]**

149. Solve $(2x^2 + 3y^2 - 7)x\,dx + (3x^2 + 2y^2 - 8)y\,dy = 0$ **[20 Marks]**

150. Test whether the equation $(x + y)^2 dx - (y^2 - 2xy - x^2)dy = 0$ is exact and hence solve it. **[20 Marks]**

151. Solve $x^3 \dfrac{d^3 y}{dx^3} + 2x^2 \dfrac{d^2 y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right)$ **[20 Marks]**

152. Determine all real valued solutions of the equations $y''' - iy'' + y' - iy = 0$, $y' = \dfrac{dy}{dx}$ **[20 Marks]**

153. Find the solution of the equation $\dfrac{d^2 y}{dx^2} + 4y = 8\cos 2x$, given that $y = 0$ and $y' = 2$ when $x = 0$ **[20 Marks]**

1994

154. Solve: $\dfrac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$ **[20 Marks]**

155. Show that if $\dfrac{1}{Q}\left(\dfrac{\partial P}{\partial y} - \dfrac{\partial Q}{\partial x}\right)$ is a function of x only, say, $f(x)$, then $F(x) = e^{\int f(x)dx}$ is an integration factor of $P\,dx + Q\,dy = 0$ **[20 Marks]**

156. Find the family of curves whose tangent from an angel $\frac{\pi}{4}$ with the hyperbola $xy = c$ **[20 Marks]**

157. Transform the differential equation $\frac{d^2y}{dx^2}\cos x + \frac{dy}{dx}\sin x - 2y\cos^3 x = 2\cos^5 x$ into one having z as independent variable where $z = \sin x$ and solve it. **[20 Marks]**

158. If $\frac{d^2x}{dt^2} + \frac{g}{b}(x-a) = 0$ (a,b and g being positive constants) and $x = a'$ and $\frac{dx}{dt} = 0$ when $t = 0$, show that $x = a + (a'-a)\cos t\sqrt{\frac{g}{b}}t$ **[20 Marks]**

159. Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x}\sin 2x$ where $D \equiv \frac{dy}{dx}$ **[20 Marks]**

1993

160. Determine the curvature for which the radius of curvature is proportional to the slope of the tangent. **[20 Marks]**

161. Show that the system of co focal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self-orthogonal. **[20 Marks]**

162. Solve $\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx + (x + \log x - x\sin y)dy = 0$ **[20 Marks]**

163. Solve $y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 = y^2$ **[20 Marks]**

164. Solve $\frac{d^2y}{dx^2} + \omega_0^2 y = a\cos\omega t$ and discuss the nature of solution as $\omega \xrightarrow{dt^2} \omega_0$ **[20 Marks]**

165. Solve $(D^4 + D^2 + 1)y = e^{-x/2}\cos\left(x\frac{\sqrt{3}}{2}\right)$ **[20 Marks]**

1992

166. By eliminating the constants a, b obtain the differential equation for which $xy = ae^x + be^{-x} + x^2$ is a solution. **[20 Marks]**

167. Find the orthogonal trajectory of the family of semi cubical parabolas $ay^2 = x^3$, where a is a variable parameter. **[20 Marks]**

168. Show that $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents hyperbolas having the following lines as asymptotes $x + y = 0$, $2x + y + 1 = 0$ **[20 Marks]**

169. Solve the following differential equation $y(1 + xy)dx + x(1 - xy)dy = 0$ **[20 Marks]**

170. Find the curves for which the portion of y- axis cut off between the origin and the tangent varies as the cube of the abscissa of the point of contact. **[20 Marks]**

171. Solve the following differential equation: $(D^2 + 4)y = \sin 2x$, given that when $x = 0$, then $y = 0$ and $\frac{dy}{dx} = 2$ **[20 Marks]**

172. Solve: $(D^3 - 1)y = xe^x + \cos^2 x$ **[20 Marks]**

173. Solve: $(x^2D^2 + xD - 4)y = x^2$ **[20 Marks]**