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Partial Differential Equations (P.D.E)

Previous year Questions from 2020 to 1992

2021-22

2020

1. Form a partial differential equation by eliminating the arbitrary functions $f(x)$ and $g(y)$ from $z = yf(x) + xg(y)$ and specify its nature (elliptic, hyperbolic or parabolic) in the region $x \geq 0, y \geq 0$

   [10 Marks]

2. Solve the partial differential equation: $(D^3 - 2D^2 D' - DD'^2 + 2D'^3)z = e^{2x+y} + \sin(x - 2y)$;

   $D = \dfrac{\partial}{\partial x}, D' = \dfrac{\partial}{\partial y}$

   [10 Marks]

3. Find the integral surface of the partial differential equation: $(x - y)y^2 \dfrac{\partial z}{\partial x} + (y - x)x^2 \dfrac{\partial z}{\partial y} = (x^2 + y^2)z$

   that contains the curve $: xz = a^3, y = 0$ on it.

   [15 Marks]

4. Find the solution of the partial differential equation: $z = \dfrac{1}{2}(p^2 + q^2) + (p - x)(q - y); \; p = \dfrac{\partial z}{\partial x}, \; q = \dfrac{\partial z}{\partial y}$

   which passes through the x -axis.

   [15 Marks]

2019

5. Form a partial differential equation of the family of surface given by the following expression.

   $\psi(x^2 + y^2 + 2z^2, y^2 - 2zx) = 0$

   [10 Marks]

6. Solve the first order quasi linear partial differential equation by the method of characteristics:

   $x \dfrac{\partial u}{\partial x} + (u - x - y) \dfrac{\partial u}{\partial y} = x + 2y$ in $x \geq 0, -\infty < y < \infty$ with $u = 1 + y$ on $x = 1$

   [15 Marks]

7. Reduce the following second order partial differential equations to canonical form and find the general

   solution: $\dfrac{\partial^2 u}{\partial x^2} - 2x \dfrac{\partial^2 u}{\partial x \partial y} + x^2 \dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial u}{\partial y} = 12x$

   [20 Marks]

2018

8. Find the partial differential equation of the family of all tangent planes to the ellipsoid $x^2 + 4y^2 + 4z^2 = 4$ which are not perpendicular to the xy plane.

   [10 Marks]

9. Find the general solution of the partial differential equation: $(y^3 x - 2x^4)p + (2y^4 - x^3 y)q = 9z(x^3 - y^3)$,

   where $p = \dfrac{\partial z}{\partial x}, q = \dfrac{\partial z}{\partial y}$, and find its integral surface that passes through the curve: $x = t, y = t^2, z = 1$.

   [15 Marks]

10. Solve the partial differential equation: $(2D^2 - 5DD' + 2D'^2)Z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$ Where

    $D \equiv \dfrac{\partial}{\partial x}, D' \equiv \dfrac{\partial}{\partial y}$.

    [15 Marks]

2017

11. Solve $(D^2 - 2DD' - D'^2)z = e^{x+2y} + x^3 + \sin 2x$ where $D \equiv \dfrac{\partial}{\partial x}, \; D' \equiv \dfrac{\partial}{\partial y}, \; D^2 \equiv \dfrac{\partial^2}{\partial x^2}, \; D'^2 \equiv \dfrac{\partial^2}{\partial y^2}$.

    [10 Marks]

12. Let Γ be a closed curve in xy-plane and let S denote the region bounded by the curve Γ. Let $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x,y)\ \forall (x,y) \in S$. If f is prescribed at each point (x,y) of S and w is prescribed on the boundary Γ of S then prove that any solution $w = w(x,y)$, satisfying these conditions, is unique. **[10 Marks]**

13. Find a complete integral of the partial differential equation $2(pq + yp + qx) + x^2 + y^2 = 0$. **[15 Marks]**

14. Reduce the equation $y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$ to canonical form and hence solve it.

[15 Marks]

15. Given the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$; $t > 0$, where $c^2 = \frac{T}{m}$, T the constant tension in the string and m is the mass per unit length of the string.
    (i)   Find the appropriate solution of the wave equation
    (ii)   Find also the solution under the conditions $y(0,t) = 0$, $y(l,t) = 0$ for all t and

$$\left[\frac{\partial y}{\partial t} \right]_{t=0} = 0,\ y(x,0) = a\sin\frac{\pi x}{t}, 0 < x < l,\ a > 0\ .$$ **[20 Marks]**

2016

16. Find the general equation of surfaces orthogonal to the family of spheres given by $x^2 + y^2 + z^2 = cz$.
[10 Marks]

17. Final he general integral of the particle differential equation $(y + zx)p - (x + yz)q = x^2 - y^2$
[10 Marks]

18. Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes though the parabola $4z + x^2 = 0$ **[15 Marks]**

19. Solve the particle differential equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$ **[15 Marks]**

20. Find the temperature $u(x,t)$ in a bar of silver of length and constant cross section of area $1cm^2$. Let density $p = 10.6g / cm^3$, thermal conductivity $K = 1.04 / (cm \sec^\circ C)$ and specific heat $\sigma = 0.056 / g^\circ C$ the bar is perfectly isolated laterally with ends kept at $0^\circ C$ and initial temperature $f(x) = \sin(0.1\pi x)^\circ C$ note that $u(x,t)$ follows the head equation $u_t = c^2 u_{xx}$ where $c^2 = k / (\rho\sigma)$
[20 Marks]

2015

21. Solve the partial differential equation: $(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$
[10 Marks]

22. Solve: $(D^2 + DD' - 2D'^2)u = e^{x+y}$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$ **[10 Marks]**

23. Solve for the general solution $p\cos(x + y) + q\sin(x + y) = z$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ **[15 Marks]**

24. Find the solution of the initial-boundary value problem

$$u_t - u_{xx} + u = 0, \qquad 0 < x < l, \; t > 0$$
$$u(0, \; t) = u(l, \; t) = 0, \quad t \geq 0$$ **[15 Marks]**
$$u(x, \; 0) = x(l - x), \qquad 0 < x < l$$

25. Reduce the second-order partial differential equation

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0 \text{ into canonical form. Hence, find its general solution}$$

[15 Marks]

2014

26. Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$ **[10 Marks]**

27. Reduce the equation $\dfrac{\partial^2 z}{\partial x^2} = x^2 \dfrac{\partial^2 z}{\partial y^2}$ to canonical form. **[15 Marks]**

28. Find the deflection of a vibrating string (length= π, ends fixed, $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$) corresponding to zero

    initial velocity and initial deflection. $f(x) = k(\sin x - \sin 2x)$ **[15 Marks]**

29. Solve $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$, given that

    (i) $u(x, \; 0) = 0, \qquad 0 \leq x \leq 1$;

    (ii) $\dfrac{\partial u}{\partial t}(x, \; 0) = x^2, \quad 0 \leq x \leq 1$

    (iii) $u(0, \; t) = u(1, \; t) = 0$, for all t **[15 Marks]**

2013

30. From a partial differential equation by eliminating the arbitrary functions f and g from
    $z = yf(x) + xg(y)$ **[10 Marks]**

31. Reduce the equation $y \dfrac{\partial^2 z}{\partial x^2} + (x + y) \dfrac{\partial^2 z}{\partial x \partial y} + x \dfrac{\partial^2 z}{\partial y^2} = 0$ to its canonical from when $x \neq y$ **[10 Marks]**

32. Solve $(D^2 + DD' - 6D'^2)z = x^2 \sin(x + y)$ where D and D' denote $\dfrac{\partial}{\partial x}$ and $\dfrac{\partial}{\partial y}$ **[15 Marks]**

33. Find the surface which intersects the surfaces of the system $z(x + y) = C(3z + 1)$, (C being a

    constant) orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$ **[15 Marks]**

34. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium
    position. If it is set vibrating by giving each point a velocity $\lambda.x(l - x)$, find the displacement of the
    string at any distance x from one end at any time t **[20 Marks]**

2012

35. Solve partial differential equation $(D - 2D')(D - D')^2 z = e^{x+y}$ **[12 Marks]**
36. Solve partial differential equation $px + qy = 3z$ **[20 Marks]**
37. A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then
    released from rest. Find the deflection $y(x, \; t)$ of the vibrating string. **[20 Marks]**

38. The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. **[20 Marks]**

2011

39. Solve the PDE $(D^2 - D'^2 + D + 3D' - 2)z = e^{(x-y)} - x^2 y$ **[12 Marks]**

40. Solve the PDE $(x + 2z)\dfrac{\partial z}{\partial x} + (4zx - y)\dfrac{\partial z}{\partial y} = 2x^2 + y$ **[12 Marks]**

41. Find the surface satisfying $\dfrac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$ **[20 Marks]**

42. Solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, $0 \le x \le a$, $0 \le y \le b$ satisfying the boundary conditions

$u(0, y) = 0$, $u(x, 0) = 0$, $u(x, b) = 0 \dfrac{\partial u}{\partial x}(a, y) = T \sin^3 \dfrac{\pi y}{a}$ **[20 Marks]**

43. Obtain temperature distribution $y(x, t)$ in a uniform bar of unit length whose one end is kept at 10^0 and the other end is insulated. Also it is given that $y(x, 0) = 1 - x$, $0 < x < 1$ **[20 Marks]**

2010

44. Solve the PDE $(D^2 - D')(D - 2D')Z = e^{2x+y} + xy$ **[12 Marks]**

45. Find the surface satisfying the PDE $(D^2 - 2DD' + D'^2)Z = 0$ and the conditions that $bZ = y^2$ when $x = 0$ and $aZ = x^2$ when $y = 0$ **[12 Marks]**

46. Solve the following partial differential equation

$zp + yq = x$
$x_0(s) = s,\ y_0(s) = 1,\ z_0(s) = 2s$

by the method of characteristics. **[20 Marks]**

47. Reduce the following 2nd order partial differential equation into canonical form and find its general solution. $xu_{xx} + 2x^2 u_{xy} - u_x = 0$ **[20 Marks]**

48. Solve the following heat equation
$u_t - u_{xx} = 0,$ $\qquad 0 < x < 2,\ t > 0$
$u(0, t) = u(2, t) = 0 \quad t > 0$ **[20 Marks]**
$u(x, 0) = x(2 - x),\qquad 0 \le x \le 2$

2009

49. Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that this equation is satisfied by the surface $yz + zx + xy = 0$. **[12 Marks]**

50. (i) Form the partial differential equation by elimination the arbitrary function f given by:

$f(x^2 + y^2, z - xy) = 0$ **[20 Marks]**

(ii)   Find the integral surface of: $x^2 p + y^2 q + z^2 = 0$ which passes through the curve: $xy = x + y, \ z = 1$ **[20 Marks]**

51.   Find the characteristics of: $y^2 r - x^2 t = 0$ where r and t have their usual meanings. **[15 Marks]**

52.   Solve: $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$ where D and D' represent $\dfrac{\partial}{\partial x}$ and $\dfrac{\partial}{\partial y}$
[15 Marks]

53.   A tightly stretched string has its ends fixed at $x = 0$ and $x = 1$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at time $t > 0$. **[30 Marks]**

2008

54.   Find the general solution of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also find the particular solution which passes through the lines $x = 1, \ y = 0$ **[12 Marks]**

55.   Find the general solution of the partial differential equation: $(D^2 + DD' - 6D'^2)z = y\cos x,$ where

$D \equiv \dfrac{\partial}{\partial x}, \ D' \equiv \dfrac{\partial}{\partial y}$ **[12 Marks]**

56.   Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0, \ x = a, \ y = 0$ and $y = b$. The edges and $x = 0, x = a$ and $y = 0$ are kept at temperature zero while the edge $y = b$ is kept at 100^0C. **[30 Marks]**

57.   Find complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method.
[15 Marks]

58.   Reduce $\dfrac{\partial^2 z}{\partial x^2} = x^2 \dfrac{\partial^2 z}{\partial y^2}$ canonical form. **[15 Marks]**

2007

59.   (i)   Form a partial differential equation by eliminating the function f from:

$z = y^2 + 2f\left(\dfrac{1}{x} + \log y\right)$

(ii)   Solve $2zx - px^2 - 2qxy + pq = 0$ **[6+6=12 Marks]**

60.   Transform the equation $yz_x - xz_y = 0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution. **[12 Marks]**

61.   Solve $u_{xx} + u_{yy} = 0$ in D where $D = \{(x,y) : 0 < x < a, 0 < y < b\}$ is a rectangle in a plane with the boundary conditions:

$u(x,0) = 0, \ u(x,b) = 0, \ 0 \le x \le a$
$u(0,y) = g(y), u_x(a,y) = h(y), \ 0 \le y \le b.$ **[30 Marks]**

62.   Solve the equation $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the conditions:

$u(0,t) = 0 = u(l,t),$ for all t and $u(x,0) = f(x)$ for all x in $[0, \ l]$ **[30 Marks]**

2006

63. Solve: $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$ **[12 Marks]**

64. Solve: $\dfrac{\partial^3 z}{\partial x^3} - 4\dfrac{\partial^3 z}{\partial x^2 \partial y} + 4\dfrac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$ **[12 Marks]**

65. The deflection of vibrating string of length l, is governed by the partial differential equation $u_{tt} = C^2 \, u_{xx}$. The ends of the string are fixed at and $x = 0$ and l. The initial velocity is zero. The initial

displacement is given by $u(x,0) = \begin{cases} \dfrac{x}{l}, & 0 < x < \dfrac{l}{2} \\ \dfrac{1}{l}(l - x), & \dfrac{l}{2} < x < l. \end{cases}$

Find the deflection of the string at any instant of time. **[30 Marks]**

66. Find the surface passing through the parabolas $z = 0$, $y^2 = 4ax$ and $z = 1$, $y^2 = -4ax$ and

satisfying the equation $x\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial z}{\partial x} = 0$ **[15 Marks]**

67. Solve the equation $p^2 x + q^2 y = z$, $p = \dfrac{\partial z}{\partial x}$, $q = \dfrac{\partial z}{\partial y}$ **[15 Marks]**

2005

68. Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the coordinate planes. **[12 Marks]**

69. Find the particular integral of $x(y - z)p + y(z - x)q = z(x - y)$ which represents a surface passing through $x = y = z$ **[12 Marks]**

70. The ends A and B of a rod 20cm long have the temperature at 30^0C and 80^0C until steady state prevails. The temperatures of ends are changed to 40^0C and 60^0C respectively. Find the temperature distribution in the rod at time t. **[30 Marks]**

71. Obtain the general solution of $(D - 3D' - 2)^2 z = 2e^{2x}\sin(y + 3x)$ where $D = \dfrac{\partial}{\partial x}$ and $D' = \dfrac{\partial}{\partial y}$

[30 Marks]

2004

72. Find the integral surface of the following partial differential equation:
$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ **[12 Marks]**

73. Find the complete integral of the partial differential equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0$, $z^2 = 4y$. **[12 Marks]**

74. Solve the partial differential equation: $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial x \partial y} - 2\dfrac{\partial^2 z}{\partial y^2} = (y - 1)e^x$ **[15 Marks]**

75. A uniform string of length l, held tightly between $x = 0$ and $x = l$ with no initial displacement, is struck at $x = a$, $0 < a < l$, with velocity v_0. Find the displacement of the string at any time $t > 0$ **[30 Marks]**

76. Using Charpit's method, find the complete solution of the partial differential equation
$p^2 x + q^2 y = z$ **[15 Marks]**

2003

77. Find the general solution of $\dfrac{\partial^2 z}{\partial x^2} + 3\dfrac{\partial^2 z}{\partial x \partial y} + 2\dfrac{\partial^2 z}{\partial y^2} = x + y + \cos(2x + 3y)$ **[12 Marks]**

78. Show that the differential equations of all cones which have their vertex at the origin are $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation. **[12 Marks]**

79. Solve $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial y^2} - 3\dfrac{\partial z}{\partial x} + 3\dfrac{\partial z}{\partial y} = xy + e^{x+2y}$ **[15 Marks]**

80. Solve the equation $p^2 - q^2 - 2px - 2qy + 2xy = 0$ using Charpit's method. Also find the singular solution of the equation, if it exists. **[15 Marks]**

81. Find the deflection $u(x,\ t)$ of a vibrating string, stretched between fixed points $(0,\ 0)$ and $(3l,\ 0)$, corresponding to zero initial velocity and following initial deflection:

$$f(x) = \begin{cases} \dfrac{hx}{l} & \text{when } 0 \le x \le 1 \\[2mm] \dfrac{h(3l - 2x)}{l} & \text{when } l \le x \le 2l \\[2mm] \dfrac{h(x - 3l)}{l} & \text{when } 2l \le x \le 3l \end{cases}$$

Where h is a constant. **[15 Marks]**

2002

82. Find two complete integrals of the partial differential equation $x^2 p^2 + y^2 q^2 - 4 = 0$ **[12 Marks]**

83. Find the solution of the equation $z = \dfrac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ **[12 Marks]**

84. Frame the partial differential equation by eliminating the arbitrary constants a and b from $\log(az - 1) = x + ay + b$ **[10 Marks]**

85. Find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = \dfrac{x}{2},\ y = 0$ **[20 Marks]**

86. Solve: $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2},\ 0 < x < l,\ t > 0$

$u(0,\ t) = u(l,\ t) = 0$

$u(x,\ 0) = x(l - x),\ \ 0 \le x \le t.$ **[30 Marks]**

2001

87. Find the complete integral partial differential equation $2p^2 q^2 + 3x^2 y^2 = 8x^2 q^2 (x^2 + y^2)$ **[12 Marks]**

88. Find the general integral of the equation $\left\{my(x + y) - nz^2\right\}\dfrac{\partial z}{\partial x} - \left\{lx(x + y) - nz^2\right\}\dfrac{\partial z}{\partial y} = (lx - my)z$

[12 Marks]

89. Prove that for the equation $z + px + qy - 1 - pqx^2y^2 = 0$ the characteristic strips are given by

$$x(t) = \frac{1}{B + Ce^{-t}}, \ y(t) = \frac{1}{A + De^{-t}}, \ z(t) = E - (AC + BD)e^{-t}$$

$$p(t) = A(B + Ce^{-t})^2, \ q(t) = B(A + De^{-t})^2 \quad \text{where } A, B, C, D \text{ and } E \text{ are arbitrary constants. Hence}$$
find the values of these arbitrary constants if the integral surface passes through the line $z = 0, \ x = y$

[30 Marks]

90. Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2 + y^2 + z^2) = C_1 y^2$ **[10 Marks]**

91. Solve the equation $x^2 \dfrac{\partial^2 z}{\partial x^2} - y^2 \dfrac{\partial^2 z}{\partial y^2} + x \dfrac{\partial z}{\partial x} - y \dfrac{\partial z}{\partial y} = x^2 y^4$ by reducing it to the equation with

constant coefficients. **[20 Marks]**

2000

92. Solve: $pq = x^m y^n z^{2l}$ **[12 Marks]**

93. Prove that if $x_1^3 + x_2^3 + x_3^3 = 1$ when $z = 0,$ the solution of the equation
$(S - x_1)p_1 + (S - x_2)p_2 + (S - x_3)p_3 = S - z$ can be given in the form
$S^3\left\{(x_1 - z)^3 + (x_2 - z)^3 + (x_3 - z)^3\right\}^4 = \left(x_1 + x_2 + x_3 - 3z\right)^3$ where $S = x_1 + x_2 + x_3 + z$ and

$p_i = \dfrac{\partial z}{\partial x_i}, \ i = 1, 2, 3.$ **[12 Marks]**

94. Solve by Charpit's method the equation $p^2 x(x - 1) + 2pqxy + q^2 y(y - 1) - 2pxz - 2qyz + z^2 = 0$

[15 Marks]

95. Solve: $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{3x+4y}.$ **[15 Marks]**

96. A tightly stretched string with fixed end points $x = 0, \ x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point x of it a velocity $kx(l - x)$, obtain at time t the displacement y at a distance x from the end $x = 0$ **[30 Marks]**

1999

97. Verify that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and find its primitive. **[20 Marks]**

98. Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1), \ c$ is constant,

orthogonally and which passes through the circle $x^2 + y^2 = 1, \ z = 1$ **[20 Marks]**

99. Find the characteristics of the equation $pq = z,$ and determine the integral surface which passes

through the passes through the parabola $x = 0, \ y^2 = z.$ **[20 Marks]**

100. Use Charpit's method to find a complete integral to $p^2 + q^2 - 2px - 2qy + 1 = 0$ **[20 Marks]**

101. Find the solution of the equation $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = e^{-x} \cos y$ which $\to 0$ as $x \to \infty$ and has the value $\cos y$

when $x = 0$ **[20 Marks]**

102. One end of a string $(x = 0)$ is fixed, and the point $x = a$ is made to oscillate, so that at time t the displacement is $g(t)$. Show that the displacement $u(x, t)$ of the point x at time t is given by

$$u(x, t) = f(ct - x) - f(ct + x) \text{ where } f \text{ is a function satisfying the relation } f(t + 2a) = f(t) - g\left(\frac{t + a}{c}\right)$$

[20 Marks]

1998

103. Find the differential equation of the set of all right circular cones whose axes coincide with the z-axis **[20 Marks]**

104. Form the differential equation by eliminating a, b and c from $z = a(x + y) + b(x - y) + abt + c$ **[20 Marks]**

105. Solve $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = xyz$ **[20 Marks]**

106. Find the integral surface of the linear partial the differential equation

   $x(y^2 + z)\dfrac{\partial z}{\partial x} - y(x^2 + z)\dfrac{\partial z}{\partial y} = (x^2 - y^2)z$ through the straight line $x + y = 0, \ z = 1$ **[20 Marks]**

107. Use Charpit's method to find a complete integral of $2x\left[\left(z\dfrac{\partial z}{\partial y}\right)^2 + 1\right] = z\dfrac{\partial z}{\partial x}$ **[20 Marks]**

108. Find a real function $V(x, y),$ which reduces to zero when $y = 0$ and satisfies the equation

   $\dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$ **[20 Marks]**

109. Apply Jacobi's method to find a complete integral of the equation

   $2x\dfrac{\partial z}{\partial x_1}x_1 x_3 + 3\dfrac{\partial z}{\partial x_2}x_3^2 + x\left(\dfrac{\partial z}{\partial x_2}\right)^2 x\dfrac{\partial z}{\partial x_3} = 0$ **[20 Marks]**

1997

110. (i) Find the differential equation of all surfaces of revolution having z-axis as the axis of rotation.

   (ii) Form the differential equation by eliminating a and b from $z = (x^2 + a)(y^2 + b)$

   [10+10=20 Marks]

111. Find the equation of surfaces satisfying $4yzp + q + 2y = 0$ and passing through

   $y^2 + z^2 = 1, \ x + z = 2$ **[20 Marks]**

112. Solve: $(y + z)p + (z + x)q = x + y$ **[20 Marks]**

113. Use Charpit's method to find complete integral of $z^2(p^2 z^2 + q^2) = 1$ **[20 Marks]**

114. Solve: $(D_x^3 - D_y^3)z = x^3 y^3$ **[20 Marks]**

115. Apply Jacobi's method to find complete integral of $p_1^3 + p_2^2 + p_3 = 1.$ Here

   $p_1 = \dfrac{\partial z}{\partial x_1}, \ p_2 = \dfrac{\partial z}{\partial x_2}, \ p_3 = \dfrac{\partial z}{\partial x_3}$ and z is a function of $x_1, x_2, x_3.$ **[20 Marks]**

1996

116. (i) differential equation of all spheres of radius λ having their center in xy-plane

   (ii) Form differential equation by eliminating f and g from $z = f(x^2 - y) + g(x^2 + y)$

   [10+10=20 Marks]

117. Solve: $z^2(p^2 + q^2 + 1) = C^2$ **[20 Marks]**

118. Find the integral surface of the equation $(x-y)y^2 p + (y-x)x^2 q = (x^2 + y^2)z$ passing through the curve $xz = a^3,\ y = 0$ **[20 Marks]**

119. Apply Charpit's method to find the complete integral of $z = px + ay + p^2 + q^2$ **[20 Marks]**

120. Solve: $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$ **[20 Marks]**

121. Find a surface passing through the lines $z = x = 0$ and $z - 1 = x - y = 0$ satisfying

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$$ **[20 Marks]**

1995

122. In the context of a partial differential equation of the first order in three independent variables, define and illustrate the terms:
    (i)   The complete integral
    (ii)   The singular integral **[20 Marks]**

123. Find the general integral of $(y+z+w)\dfrac{\partial w}{\partial x} + (z+x+w)\dfrac{\partial w}{\partial y} + (x+y+w)\dfrac{\partial w}{\partial z} = x + y + z$ **[20 Marks]**

124. Obtain the differential equation of the surfaces which are the envelopes of a one-parameter family of planes. **[20 Marks]**

125. Explain in detail the Charpit's method of solving the nonlinear partial differential equation

$$f\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0$$ **[20 Marks]**

126. Solve $\dfrac{\partial z}{\partial x_1}\dfrac{\partial z}{\partial x_2}\dfrac{\partial z}{\partial x_3} = z^3 x_1 x_2 x_3$ **[20 Marks]**

127. Solve $(D_x^{\ 3} - 7D_x D_y^{\ 2} - 6D_y^{\ 3})z = \sin(x + 2y) + e^{3x+y}$ **[20 Marks]**

1994

128. Find the differential equation of the family of all cones with vertex at $(2, -3, 1)$ **[20 Marks]**

129. Find the integral surface of $x^2 p + y^2 q + z^2 = 0,\ p \equiv \dfrac{\partial z}{\partial x},\ q \equiv \dfrac{\partial z}{\partial y}$ which passes through the hyperbola

$xy = x + y,\ z = 1$ **[20 Marks]**

130. Obtain a Complete Solution of $pq = x^m y^n z^{2l}$ **[20 Marks]**

131. Use the Charpit's method to solve $16p^2 z^2 + 9q^2 z^2 + 4z^2 - 4 = 0$. Interpret geometrically the complete solution and mention the singular solution. **[20 Marks]**

132. Solve $(D^2 + 3DD' + 2D'^2)z = x + y$, by expanding the particular integral in ascending powers of D, as well as in ascending powers of D'. **[20 Marks]**

133. Find a surface satisfying $(D^2 + DD')z = 0$ and touching the elliptic paraboloid $z = 4x^2 + y^2$ along its section by the plane $y = 2x + 1$. **[20 Marks]**

1993

134. Find the surface whose tangent planes cut off an intercept of constant length R from the axis of z.
[20 Marks]

135. Solve $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2(x^2 + y^2)z$
[20 Marks]

136. Find the integral surface of the partial differential equation $(x - y)p + (y - x - z)q = z$ through the circle $z = 1, \ x^2 + y^2 = 1$
[20 Marks]

137. Using Charpit's method find the complete integral of $2xz - px^2 - 2qxy + pq = 0$
[20 Marks]

138. Solve $r - s + 2q - z = x^2y^2$
[20 Marks]

139. Find the general solution of $x^2r - y^2t + xp - yq = \log x$
[20 Marks]

1992

140. Solve:
$$(2x^2 - y^2 + z^2 - 2yz - zx - xy)p + (x^2 + 2y^2 + z^2 - yz - 2zx - xy)q = (x^2 + y^2 + 2z^2 - yz - zx - 2xy)$$
[20 Marks]

141. Find the complete integral of $(y - x)(qy - px) = (p - q)^2$
[20 Marks]

142. Use Charpit's method to solve $px + qy = z\sqrt{1 + pq}$
[20 Marks]

143. Find the surface passing through the parabolas $z = 0, \ y^2 = 4ax; \ z = 1, \ y^2 = -4ax$ and satisfying the differential equation $xr + 2p = 0$
[20 Marks]

144. Solve: $r + s - 6t = y\cos x$
[20 Marks]

145. Solve: $\dfrac{\partial^2 u}{\partial x^2}\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial z}{\partial y} - z = \cos(x + 2y) + e^y$
[20 Marks]