ANALYTICAL GEOMETRY

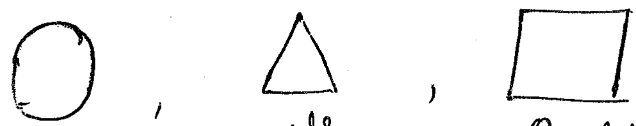
Syllabus :- Paper 1, Section A.

Cartesian and Polar co-ordinates in 3 dimensions, Second degree eqn's in three variables, reduction to canonical forms, straight lines, Shortest distance b/w 2 skew lines; Plane, Sphere, cone, cylinder, paraboloid, ellipsoid, hyperboloid of one & 2 sheets & their properties.

PLANES :-

A plane is defined as the surface which is such that the line joining any 2 points on it lies completely on it

Eg:



Circle, $\triangle^{le}$, Quadrilateral

general form of 1^{st} degree eqn in x, y, z gives plane eqn i.e

$P = ax + by + cz + d = 0 \rightarrow$ General form of plane eqn.

Note points

* If plane passes through origin $O(0,0,0)$

$$a(0) + b(0) + c(0) + d = 0$$
$$d = 0$$

$$ax + by + cz + d = 0$$
$$ax + by + cz + 0 = 0$$
$$\boxed{ax + by + cz = 0}$$

* If plane passes through points (α, β, γ)

$$a\alpha + b\beta + c\gamma + d = 0$$
$$d = -(a\alpha + b\beta + c\gamma)$$

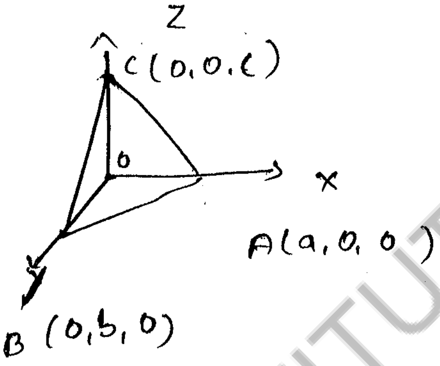
$$ax + by + cz + d = 0$$
$$ax + by + cz - (a\alpha + b\beta + c\gamma) = 0$$
$$\boxed{a(x-\alpha) + b(y-\beta) + c(z-\gamma) = 0}$$

Eg. $(1,2,3)$

$$a(x-1)+b(y-2)+c(z-3)=0$$

→ Intercept form of the plane eqⁿ :-

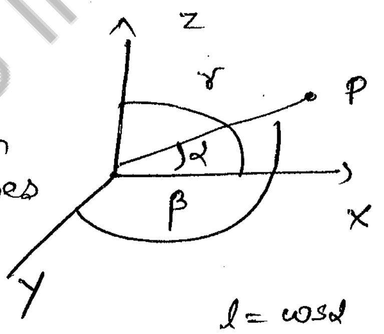
$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$



→ Normal form of the plane eqⁿ :-

$$lx+my+nz=p$$

$l^2+m^2+n^2=1$ → Direction cosines



p is perpendicular distance to the origin from plane

$l=\cos\alpha$

$m=\cos\beta$

$n=\cos\gamma$

→ Can be converted from one form to another. general to normal form

$$ax+by+cz+d=0$$

$$ax+by+cz=-d$$

$$\left(\frac{-a}{\sqrt{\Sigma a^2}}\right)x+\left(\frac{-b}{\sqrt{\Sigma a^2}}\right)+\left(\frac{-c}{\sqrt{\Sigma a^2}}\right)=\frac{|d|}{\sqrt{\Sigma a^2}}$$

$$\sqrt{\Sigma a^2}=\sqrt{a^2+b^2+c^2}$$

DCS ⇒ $l=\frac{-a}{\sqrt{\Sigma a^2}}$, $m=\left(\frac{-b}{\sqrt{\Sigma a^2}}\right)$, $n=\left(\frac{-c}{\sqrt{\Sigma a^2}}\right)$

Direction ratios $-a,-b,-c$ i.e a,b,c

$\boxed{\text{⊥lar distance from origin to plane } p=\frac{|d|}{\sqrt{\Sigma a^2}}}$

→ $ax+by+cz=-d$ → General form to intercept

$$\frac{x}{(-d/a)}+\frac{y}{(-d/b)}+\frac{z}{(-d/c)}=1$$

→ Normal to intercept form

$$\frac{x}{(p/l)}+\frac{y}{(p/l)}+\frac{z}{(p/l)}=1$$

Eg. if Drs are 1, 2, 3 = $\sqrt{1+4+9}$ (sum of root of square) $\neq 1$

dcs are $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

$\boxed{\text{Drs} \neq 1}$ for $\boxed{\text{Dcs} = 1 \text{ (sum of squares)}}$

Drs $-a, -b, -c$ is same as a, b, c (as viewed from 180°)

Dcs of x axis $(1, 0, 0)$

—"— y-axis $(0, 1, 0)$

—"— z-axis $(0, 0, 1)$

Eg. $\frac{l}{a}=\frac{m}{b}=\frac{n}{c}=\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}=\frac{1}{\sqrt{a^2+b^2+c^2}}=\frac{1}{\sqrt{\Sigma a^2}}$

$l=\frac{a}{\sqrt{\Sigma a^2}}, \quad m=\frac{b}{\sqrt{\Sigma a^2}}, \quad n=\frac{c}{\sqrt{\Sigma a^2}}$

l, m, n are Dcs

a, b, c are Dr's

If a, b, c & α, β, γ are fixed

$a(x-\alpha) + b(y-\beta) + c(z-\gamma) = 0 \rightarrow$ Is only one plane

if it is not fixed than may have many plane.

Questions :-

(1) A variable plane moves show that the sum of reciprocals of intercepts on 3 co-ordinate axis is constant, & P.T it passes through a fixed point.

Soln : Intercept form of plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad -(1), \qquad a, b, c \text{ are } x, y, z \text{ intercepts}$$

sum of reciprocal in problem is constant $\quad 1/a + 1/b + 1/c = 1/K$

$(K \neq 0)$

let constant be $1/K$

$$\frac{K}{a} + \frac{K}{b} + \frac{K}{c} = 1$$

So the plane (1) passes through points (K, K, K)

Hence proved.

(2) A plane meets the coordinate axis A, B, C such that the centroid of $\triangle^{le}$ ABC is the point (p, q, r). Then S.T the eqn of the plane $\frac{x}{p} + y/q + z/r = 3$

Soln: Intercept form of eqn

$$x/a + y/b + z/c = 1$$

a, b, c are x, y, z intercepts

$A(a, 0, 0) \quad B(0, b, 0) \quad C(0, 0, c)$