Syllabus → Dynamics

Paper - 1 . Section - B

{Rectilinear motion, Simple Harmonic motion, Motion in a plane, projectiles, constrained motion, work & Energy, conservation of Energy, Kepler law's, Orbits under central forces} → Dynamics.

{Equilibrium of s/m of particles, work & potential energy, Friction common catenary. principle of virtual work. Stability of equilibrium, equilibrium of forces in 3 dimensions} → Statics.

Rectilinear motion :-

When a particle (point) moves along a straight line then its motion is called Rectilinear motion.

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 - u^2 = 2as$

displacement $= \frac{dx}{dt}\hat{i}$

Velocity $= \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$= \frac{d^2x}{dt^2}$

(1) If at time 't' the displacement 'x' of a particle moving away from the origin is given by $x = a\sin t + b\cos t$ find the velocity & acceleration of the particle

Sol^n

$$V = \frac{dx}{dt} = \frac{d}{dt}(a\sin t + b\cos t)$$

$$= a\cos t - b\sin t$$

$$a = \frac{dv}{dt} = -a\sin - b\cos t = -[a\sin t + b\cos t]$$

$$a = -x$$

$$\frac{d^2x}{dt^2} = -x$$

$$\frac{d^2x}{dt^2} + x = 0$$

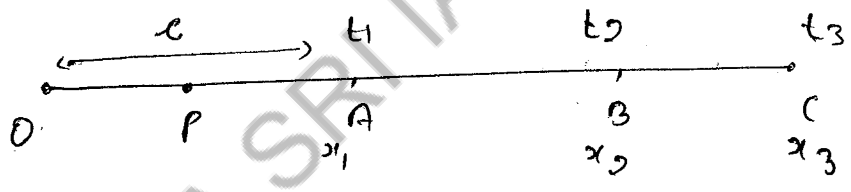
(2) A particle moves in a straight line with constant acceleration & its distance from the origin 'O' on the line [not necessarily the position at time $t=0$] at times t_1, t_2, t_3 are x_1, x_2, x_3 respectively. S.T if t_1, t_2, t_3 are in AP & whose common difference is d. & x_1, x_2, x_3 are in GP then the acceleration is $\dfrac{(\sqrt{x_1} - \sqrt{x_3})^2}{d^2}$

Soln

if t_1, t_2 & t_3 are in AP then $2t_2 = t_1 + t_3$

if x_1, x_2, x_3 are in GP then $x_2^2 = x_1 x_3$

$$S = ut + \tfrac{1}{2}at^2$$



$OA = x_1, \quad OB = x_2, \quad OC = x_3 \qquad OP = c$

$$S - c = ut_1 + \tfrac{1}{2}at_1^2$$

$$S = c + ut_1 + \tfrac{1}{2}at_1^2$$

$$x_1 = ut_1 + \tfrac{1}{2}at_1^2 \quad -(1) \text{ 11}^{ly}$$

$$x_2 = ut_2 + \tfrac{1}{2}at_2^2 \quad -(2)$$

$$x_3 = ut_3 + \tfrac{1}{2}at_3^2 \quad -(3)$$

$S - c = x_1$

11^{ly} $S - c = x_2$

$S - c = x_3$

$x_2^2 = x_1 x_3$, $\quad 2t_2 = t_1 + t_3$

(1) + (3) − 2(2)

$\rightarrow x_1 + x_3 - 2x_2 = ut_1 + ut_3 - 2ut_2 + \frac{1}{2}at_1^2 + \frac{1}{2}at_3^2 - 2(\frac{1}{2})at_2^2$

$$x_1 + x_3 - 2\sqrt{x_1 x_3} = u(t_1 + t_3 - 2t_2) + \frac{1}{2}a(t_1^2 + t_3^2 - 2t_2^2)$$

$$x_1 + x_3 - 2\sqrt{x_1 x_3} = u(0) + \frac{1}{2}a\left(t_1^2 + t_3^2 - 2\left(\frac{1}{4}\right)(t_1^2 + t_3^2 + 2t_1t_3)\right)$$

$$x_1 + x_3 - 2\sqrt{x_1 x_3} = \frac{1}{2}a\left[\frac{1}{2}t_1^2 + \frac{1}{2}t_3^2 - t_1t_3\right]$$

$2t_2 = t_1 + t_3$

$t_2 = \frac{1}{2}(t_1 + t_3)$

$t_2^2 = \frac{1}{4}(t_1 + t_3)^2$

$$(\sqrt{x_1} - \sqrt{x_2})^2 = \frac{1}{2}a\left[\frac{1}{2}t_1^2 + \frac{1}{2}t_3^2 - t_1t_3\right]$$

$$= \frac{1}{2} \times \frac{1}{2}a\left[t_1^2 + t_3^2 - 2t_1t_3\right]$$

$$= \frac{1}{4}a\left[t_1 - t_3\right]^2$$

In AP

$t_2 - t_1 = d$

$t_3 - t_2 = d$

$t_3 - t_1 = 2d$

$$= \frac{1}{4}a[2d]^2$$

$$= \frac{1}{4}a(4d^2)$$

$$(\sqrt{x_1} - \sqrt{x_2})^2 = ad^2$$

$$a = \left[\frac{(\sqrt{x_1} - \sqrt{x_2})^2}{d^2}\right]$$

Newton's law's of motion :-

1st law :- Everybody continues in a state of rest or of uniform motion in a straight line unless it is compelled by some external force (forces) to change its state

2nd law :- The rate of change of the momentum of a body is proportional to impressed force & takes place in the direction in which the force acts.

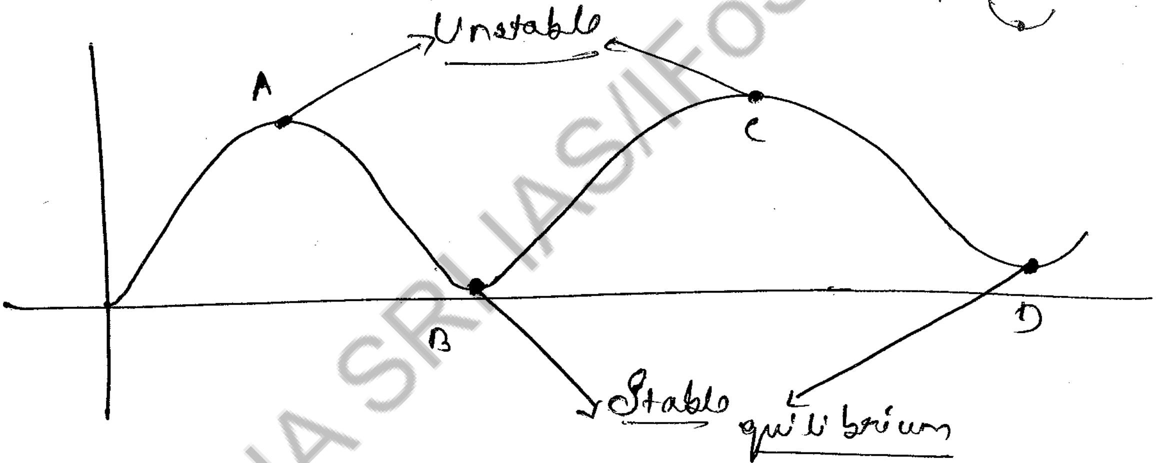
$$f = \frac{dp}{dt} \Rightarrow f = ma$$

3rd law :- To every action there is a equal & opposite reaction

STATICS

Equilibrium of s/m of particles, work and potential energy, friction; common catenary. Principle of virtual work, Stability of equilibrium, Equilibrium of forces in 3 dimensions.

⇒ Stable and Unstable equilibrium :-

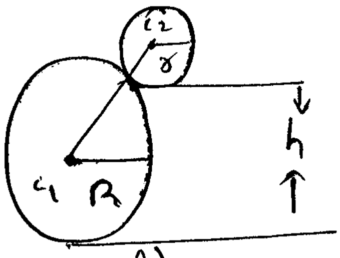


* A body rests in equilibrium upon another fixed body, the portions of 2 bodies in contact being spheres of radii R & r respectively & straightline joining the centres of the spheres being vertical ; If the 1st body is slightly displaced then

If $\frac{1}{h} > \frac{1}{r} + \frac{1}{R}$ then stable equilibrium

$\frac{1}{h} < \frac{1}{r} + \frac{1}{R}$ then unstable equilibrium

$\frac{1}{h} = \frac{1}{r} + \frac{1}{R}$ then also unstable equilibrium



(fixed)

* A body rests in equilibrium inside another concave fixed body, the portions of 2 bodies in contact being spheres of radii r and R respectively and straight line joing centres being vertical if the 1^{st} body slightly displaced then

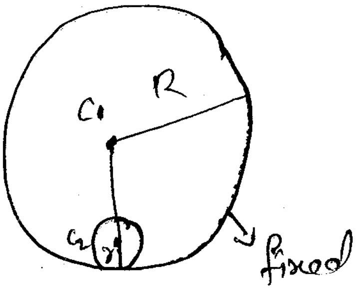
$\frac{1}{h} > \frac{1}{r} - \frac{1}{R}$ then stable equilibrium

$\frac{1}{h} < \frac{1}{r} - \frac{1}{R}$ then unstable equilibrium

$\frac{1}{h} = \frac{1}{r} - \frac{1}{R}$ Stable or unstable equilibrium or

$R >$ or $< 2r$

S.E U.S.E



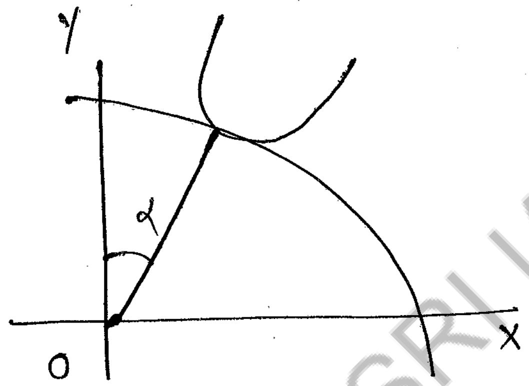
$R < 2r$ (U.S.E)

$R > 2r$ (S.E)

* A body rests in equilibrium upon another fixed body which is fixed & the portions of 2 bodies in contact have radius of the curvature ρ_1 & ρ_2 the "centre of gravity" of the 1st body is at height 'h' above the point of contact and the common normal makes an angle 'α' with the verticle then

- $h < \dfrac{\rho_1 \rho_2}{\rho_1 + \rho_2} \cos\alpha \rightarrow$ Stable equilibrium (S.E)

- $h > \dfrac{\rho_1 \rho_2}{\rho_1 + \rho_2} \cos\alpha \rightarrow$ Unstable equilibrium (U.S.E)



if $h = \dfrac{\rho_1 \rho_2}{\rho_1 + \rho_2}$ it fails

$$\frac{d}{ds}\left(1/\rho_1\right) + \frac{d}{ds}\left(1/\rho_2\right) \underset{S.E}{<0} \text{ or } \underset{U.S.E}{>0}$$

if it also fails

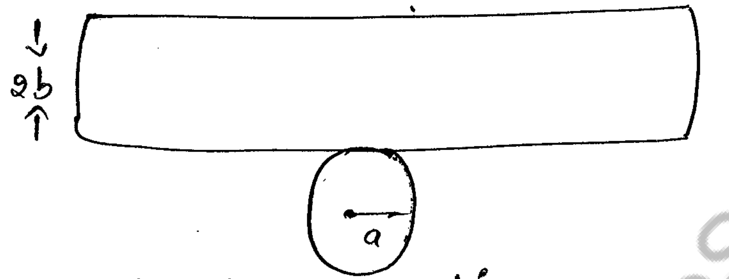
$$\frac{d^2}{ds^2}\left(1/\rho_1\right) + \frac{d^2}{ds^2}\left(1/\rho_2\right) + \frac{(\rho_1+\rho_2)(\rho_1+2\rho_2)}{\rho_1^3 \rho_2^2}$$

* Energy test for stability :-

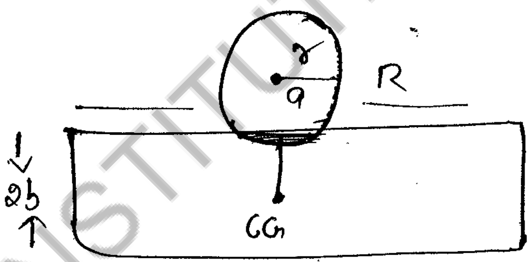
The potential energy is minimum for S.E ~~for~~ maximum for unstable equilibrium (U.S.E)

① A uniform ~~being~~ beam, of thickness 2b rests symmetrically [4] on a perfectly rough horizontal cylinder of radius 'a'. S.T the equilibrium beam will be stable or unstable according as 'b' less than or greater than 'a' [$b<a$ S.E, $b>a$ U.S.E]

Soln :



According to question



Actual question

here h_{CG} (distance) $= b$

$r=a$, $R=\infty$ (because no length specified for beam)

$h=b$

$\frac{1}{h} < \frac{1}{r} + \frac{1}{R}$ (U.S.E)

$1/b < \frac{1}{a} + \frac{1}{\infty}$

$1/h < \frac{1}{a} + 0$

$1/h < 1/a \Rightarrow b>a$

$\boxed{b<a \rightarrow S.E}$

$\frac{1}{h} > 1/r + \frac{1}{R}$ (S.E)

$\frac{1}{b} > \frac{1}{a} + \frac{1}{\infty}$

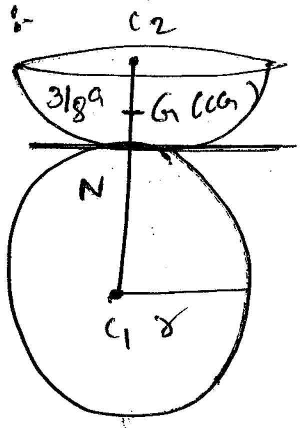
$\frac{1}{b} > 1/a$

$1/b > 1/a \Rightarrow b<a$

$\boxed{b>a \text{ USE}}$

(2) A hemisphere rests in equilibrium on a sphere of equal radius; S.T equilibrium is unstable when the curved surface rest on the sphere. and stable when the flat surface on the hemisphere rest on the sphere

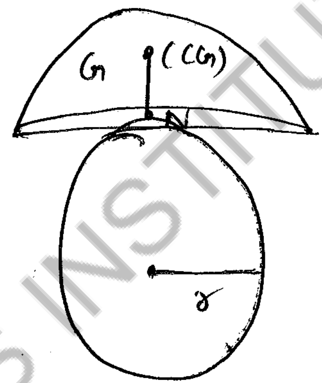
Soln :- case (I)



Un Stable

$r = R = a$

case (II)



Stable

Centre of gravity for ~~centre~~ circle = r

Centre of gravity for hemisphere = $\frac{3r}{8}$

case (I) When the curved surface rest on a sphere. w.r.t

(CG) of (Centre of gravity) hemisphere = $\frac{3}{8}a$ [$r = R = a$ equal radius

CG of hemisphere 3/8 (radius) from the centre

$$h = GN = CN - CG = a - \frac{3}{8}a = \frac{5a}{8}$$

$$h = 5a/8$$

$$1/h = \frac{8}{5a} \quad -(1)$$