LPP

Linear Programming

Paper - 2
Section - A

Min. marks - 25
Max. marks - 60

Syllabus

- Linear programming problems, Basic solution, Basic feasible solution and optimal solution, Graphical method and simplex method solutions, Duality, Transportation and Assignment Problems.

Assignment Problems

(1) Solve the minimum assignment problem [Hungarian Method]

man → 1 2 3 4

Job ↓

	1	2	3	4
I	12	30	21	15
II	18	33	9	31
III	44	25	24	21
IV	23	30	28	14

Step 1 :- Substract with least number in row

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Step 2 :- Substract with least number in column after step 1.

0	14	9	3
9	20	0	22
23	0	3	0
9	12	14	0

Step 3 :- Assign with the single least number and strike if the number is present more than one in a column.

$\boxed{0}$	14	9	3
9	20	$\boxed{0}$	22
23	$\boxed{0}$	3	$\cancel{0}$
9	12	14	$\boxed{0}$

Single zero rows,
Single zero column
Double zero rows,
Double zero column,

I - 1 12

II - 3 9

III - 2 25

IV - 4 14

12 + 9 + 25 + 14 => 60

② Solve the minimum Assignment problem whose cost effective matrix is

	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

*

0	1	2	3
0	1	2	3
0	1	2	1
0	2	5	1

*

0	0	0	2
0	0	0	2
0	0	0	0
0	1	3	0

*

$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	2
$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$
$\boxed{0}$	1	3	$\cancel{0}$

A - II   3
B - III   6
C - IV   8
D - I   3

$3 + 6 + 8 + 3 = 20$

(3)

	I	II	III	IV	V	VI
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

$$\begin{array}{r} 78 \\ 43 \\ \hline 35 \end{array} \qquad \begin{array}{r} 74 \\ 27 \\ \hline 47 \end{array}$$

(1)

0	13	49	2	10	18
0	35	29	7	20	5
13	0	63	9	17	5
47	15	0	22	12	5
25	0	46	11	14	7
0	53	50	28	14	25

(2)

⊗	13	49	$\boxed{0}$	⊗	13	
⊗	35	29	5	10	$\boxed{0}$	✓ (4)
13	⊗	63	7	7	⊗	✓ (1)
47	15	$\boxed{0}$	20	2	⊗	
25	$\boxed{0}$	46	9	4	2	✓ (5)
$\boxed{0}$	53	50	26	4	20	✓ (7)
✓ (6)	✓ (3)				✓ (2)	

(1) No assignment row (✓)

⊗ columns (✓)

$\boxed{0}$ rows (✓)

Draw lines on Ticked columns & Non Ticked rows.

Junction

0	13	49	0	0	17
0	35	29	5	10	0
13	0	63	7	7	0
47	15	0	20	2	0
25	0	46	9	4	2
0	53	50	26	4	20

Add the minimum number to the junction in the non passing line substract with least number

Here 4 is the minimum number so substract from other and add at Junction

Now Assign

4	17	49	$\boxed{0}$	~~0~~	17
$\boxed{0}$	35	25	1	6	~~0~~
13	~~0~~	59	3	3	$\boxed{0}$
51	19	$\boxed{0}$	20	2	4
25	$\boxed{0}$	42	5	~~0~~	2
~~0~~	53	46	22	$\boxed{0}$	20

A	IV	11
B	I	43
C	VI	33
D	III	27
E	II	11
F	IV	17
		142

4	17	49	$\boxed{0}$	~~0~~	17
~~0~~	35	25	1	6	$\boxed{0}$
13	$\boxed{0}$	59	3	3	~~0~~
51	19	$\boxed{0}$	20	2	4
25	~~0~~	42	5	$\boxed{0}$	2
$\boxed{0}$	53	46	22	~~0~~	20

A - 11
B - 48
C - 28
D - 27
E - 25
F - 3

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COMPLEX ANALYSIS

Paper-2, Section A   Min 25m, Max 55 marks

Syllabus :- Analytic function, Cauchy Riemann Equations, Cauchy's Integral formula, Power series representation of analytical functions, Taylor series, Singularities, Laurent Series, Cauchy's residual theorem, contour integration.

$$C = \{ z = a+ib \mid a, b \in R, \ i^2 = -1, \ i = \sqrt{-1} \}$$

Real number = (Rational + Irrational) number

$(-\infty, \infty)$

$e^{i\theta} = \cos\theta + i\sin\theta$

$\cosh x = \dfrac{e^{x}+e^{-x}}{2}$

$\sinh x = \dfrac{e^{x}-e^{-x}}{2}$

$\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$

$\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$

$\sin(a+b) = \sin a \cos b . \cos a \sin b$

$z = x+iy = re^{i\theta}$

$x = r\cos\theta$

$y = r\sin\theta$

$\theta = \tan^{-1}(y/x)$

$\boxed{w = u+iv}$

$\dfrac{1}{i} = -i$

$-\dfrac{1}{i} = i$

$e^{x} = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dots$

$\cos(i\theta) = \cosh\theta$

$\sin(i\theta) = +i\sinh\theta$

Analytic Function :-

Neighbourhood of a point [(nbd) of a point], Deleted nbd, limits of a function, Continuity of a function, derivability of a function, Integrability of a function

For real analysis

a

Nbd of a $\quad (a-\delta,\ a+\delta) \Rightarrow |x-a| < \delta$

Deleted nbd $\quad (a-\delta,\ a+\delta) \Rightarrow 0 < |x-a| < \delta$

" Removing a !!

Limit of a function

$$\lim_{x \to a} f(x) = l$$

$$\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x) = l$$

if $\lim_{x \to a} f(x) = f(a)$ then the function is continuous at 'a'

$$\left.\frac{dy}{dx}\right|_{x=a} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = f'(a) \rightarrow$$ It is derivable at a

In complex variable

$$|z - z_1| < \delta \rightarrow \text{nbd}$$

$$|(x+iy) - (x_1+iy_1)| < \delta$$

$$|(x-x_1) + i(y-y_1)| < \delta$$

$$(x-x_1)^2 + (y-y_1)^2 < \delta^2 \rightarrow \text{Disc}$$

If z_1 is equal to δ^2 than it is circle

$$\left.\begin{aligned} 0 < |z - z_1| < \delta \\ |z - z_1| < \delta_1 \quad z \neq z_1 \end{aligned}\right\} \text{Deleted nbd}$$

$$\lim_{z \to z_1} f(z) = l \rightarrow \text{limit}$$

$$\lim_{z \to z_1} f(z) = l = f(z_1)$$

then the function is continuous

~~f(z)~~

$$f'(z_1) = f'(z)\Big]_{z=z_1} = \lim_{z \to z_1} \frac{f(z) - f(z_1)}{z - z_1} \rightarrow \text{Derivative}$$

Analytic, Holomorphic & Regular functions

① **Analytic** :- let $f(z)$ be the single valued function defined in a domain 'D' then $f(z)$ said to be analytic at point z_0 of D if it is differentiable not only at z_0 but also in some nbd of z_0.

(i) $$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$ if it exists

(ii) & some nbd of z_0 (disc) is also differentiable

Note:- * A single valued function which is differentiable at each point of a domain D is said to analytic at the domain D

* A function which is analytic is called Holomorphic function

* Isolated Singularity of $f(z)$:- A function $f(z)$ is analytic at some point in every neighborhood of a point z_0 except z_0 itself is called isolated singularity.

② **Regular function** :-

A function $f(z)$ is said to be regular at a point if it has removable singularity at z_0

⇒ Necessary & Sufficient condn for $f(z)$ to be analytic :-

(a) **Necessary Condn** :-

A function $f(z) = u(x,y) + iv(x,y)$ is differentiable at any point $z = x+iy$ & the partial derivatives u_x, v_x, u_y, v_y should exist & satisfy C-R equations

C-R eqⁿ are

$U_x = V_y$ & $U_y = -V_x$

$$\left[\frac{\delta u}{\delta x} = \frac{\delta v}{\delta y} \quad \& \quad \frac{\delta u}{\delta y} = -\frac{\delta v}{\delta x}\right]$$

(b) Sufficient condⁿ:

The single valued continous function $f(z)$ is analytic in a domain D if 4 partial derivatives exists $[u_x, u_y, v_x, v_y]$ are continuous and satisfy C-R equation.

$$\frac{\delta u}{\delta x} = \lim_{x \to 0} \frac{u(x,0) - u(0,0)}{x}$$

$$\frac{\delta u}{\delta y} = \lim_{y \to 0} \frac{u(0,y) - u(0,0)}{y}$$

(1) S.T $f(z) = \sin x \cosh y + i \cos x \sinh y$ is analytic everywhere.

Solⁿ: $u = \sin x . \cosh y$ $\quad v = \cos x . \sinh y$

w.k.T $\sin x$ & $\cos x$ are continuous & $\sinh x$ & $\cosh x$ are continuous everywher, we also know that product of 2 function is continuous everywhere

$\therefore$ $u = \sin x . \cosh y$, $v = \cos x . \sinh y$ are continuous everywher

$w = f(z) = u + iv$ is continuous everywhere

$u_x = \frac{\delta u}{\delta x} = \cosh y \cos x$

$u_y = \sin x \sinh y$

$v_x = -\sin x . \sinh y$

$v_y = \cos x . \cosh y$

VECTOR ANALYSIS

Syllabus

Scalar & Vector fields, Differentiation of vector field of a scalar variable, Gradient, divergence, & curl in Cartesian & Cylindrical co-ordinates; Higher Order derivatives, Vector identities & vector equations, Application to geometry, Curves in space, Curvature & Torsion, Serret-Frenet's Formulae, Gauss & Stokes theorem, Green's identities.

$f(x, y, z) = x^3 y^2 z$ → Scalar point Function (SPF)

$f(x, y, z) = x^3 i + y^2 j + zK$ → Vector point function. (VPF)

If a, b, c are differentiable VPF of scalar variable t

ϕ is differentiable SPF of the same variable t, then

1. $\dfrac{d}{dt}(a \pm b) = \dfrac{da}{dt} \pm \dfrac{db}{dt}$

2. $\dfrac{d}{dt}(a \cdot b) = a \cdot \dfrac{db}{dt} + \dfrac{da}{dt} \cdot b$

3. $\dfrac{d}{dt}(a \times b) = a \times \dfrac{db}{dt} + \dfrac{da}{dt} \times b$

4. $\dfrac{d}{dt}(\phi a) = \phi \dfrac{da}{dt} + \dfrac{d\phi}{dt} \cdot a$

5. $\dfrac{d}{dt}[a \; b \; c] = \left[\dfrac{da}{dt} \; b \; c\right] + \left[a \; \dfrac{db}{dt} \; c\right] + \left[a \; b \; \dfrac{dc}{dt}\right]$

6. $\dfrac{d}{dt}\left[a \times (b \times c)\right] = \dfrac{da}{dt} \times (b \times c) + a \times \left(\dfrac{db}{dt} \times c\right) +$

$a \times \left(b \times \dfrac{dc}{dt}\right)$

$\Rightarrow a \cdot b = b \cdot a$

$\Rightarrow a \times b = -(b \times a)$

Cross product of same vectors are zero

$\vec{a} \times \vec{a} = 0$

$$\vec{a} \times \vec{a} = \begin{vmatrix} i & j & K \\ a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

Derivative of a function of a function :

Suppose $\vec{r}$ is the differentiable vector function of a scalar variable 's' & 's' is the differentiable scalar function of scalar variable 't' then $\vec{r}$ is a function of a function or composite func. is called function of a func.

$$\boxed{\underset{\text{Vector}}{\frac{dr}{dt}} = \underset{\text{vector}}{\frac{dr}{ds}} \; \underset{\text{Scalar}}{\frac{ds}{dt}}}$$

$$\left|\frac{dr}{dt}\right| = \left|\frac{dr}{ds}\right| \frac{ds}{dt}$$

Note

1. $a(t)$ be constant vector $\iff \frac{da}{dt} = 0$
2. If a differentiable VPF of scalar variable 't' and if $|\vec{a}| = a$

   (a) $\frac{d}{dt}(a^2) = 2a \frac{da}{dt}$

   (b) $\vec{a} \cdot \frac{d\vec{a}}{dt} = a \frac{da}{dt}$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\vec{a} \cdot \vec{a} = a^2$

$\vec{a} \cdot \frac{d\vec{a}}{dt} + \frac{d\vec{a}}{dt} \cdot \vec{a}$

$= 2a \frac{da}{dt}$

$\cancel{2}\left[\vec{a} \cdot \frac{d\vec{a}}{dt}\right] = \cancel{2} a \frac{da}{dt}$

$\vec{a} \cdot \frac{d\vec{a}}{dt} = a \frac{da}{dt}$

(c) If a has a constant length [fixed magnitude] then

$\vec{a} \cdot \frac{d\vec{a}}{dt}$ are perpendicular provided

$$\left| \frac{d\vec{a}}{dt} \right| \neq 0$$

(d) If $\vec{a}(t)$ have constant magnitude then $\Leftrightarrow$ $a \cdot \frac{da}{dt} = 0$

(or)

$$\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$$

(e) $$\frac{d}{dt}\left(\vec{a} \times \frac{d\vec{a}}{dt}\right) = \frac{d\vec{a}}{dt} \times \frac{d\vec{a}}{dt} + \vec{a} \times \frac{d^2\vec{a}}{dt^2}$$

$$= \vec{0} + \vec{a} \frac{d^2\vec{a}}{dt^2}$$

$$\frac{d}{dt}\left(\vec{a} \times \frac{d\vec{a}}{dt}\right) = \vec{a} \frac{d^2\vec{a}}{dt^2}$$

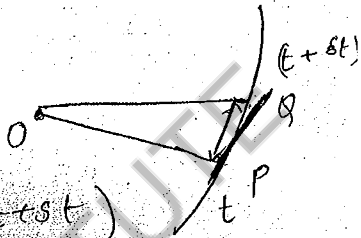
(f) $\vec{a}(t)$ have constant direction $\Leftrightarrow$ $\vec{a} \times \frac{d\vec{a}}{dt} = \vec{0}$

Space Curves

It involves 3 dimensional plane

Geometrical Significance of $\frac{dr}{dt}$ and Unit tangent vector to a curve :-

$\frac{dr}{dt}$ is a vector parallel to the tangent at 'P' of the curve $r = f(t)$



$$\vec{OP} = \vec{r} = r(t)$$

$$\vec{OQ} = \vec{r} + \delta\vec{r} = r(t+\delta t)$$

$$\vec{PQ} = \vec{OQ} - \vec{OP} = \delta\vec{r}$$

$$\vec{PQ} = \delta\vec{r} = r(t+\delta t) - r(t)$$

$$\lim_{\delta t \to 0} = \lim_{\delta t \to 0} \frac{r(t+\delta t) - r(t)}{\delta(t)}$$

$$\frac{dr}{dt} = r'(t) \rightarrow \text{tangent}$$

t = Unit tangent vector $= \frac{dr}{ds}$

$$\boxed{t = \frac{dr}{ds}}$$

$$\left|\frac{d\vec{r}}{dt}\right| = \left|\frac{d\vec{r}}{ds}\frac{ds}{dt}\right|$$

$$\left|\frac{d\vec{r}}{dt}\right| = \left|\frac{d\vec{r}}{ds}\right| \frac{ds}{dt}$$

$$\left|\frac{d\vec{r}}{dt}\right| = |t| \frac{ds}{dt}$$

$|t| =$ Unit vect

$$\frac{ds}{dt} = \left|\frac{d\vec{r}}{\ }\right|$$

NUMERICAL ANALYSIS

Paper- 2 (max 96, min 70)
Section - B

Syllabus :- Numerical Methods :- Solution of Algebraic and transcedental eq^n of one variable by Bisection, Regular-Falsi and Newton-Raphson method.
Solutions of system of linear equation by Gaussian elimination, Gauss-Jordan (direct), Gauss-Siedal iterative methods, Newton's (forward and Backward) Interpolation, Legrenge's interpolation, Numerical Integration :- Trapezoidal Rule, Simpson's Rules, ($1/3^{rd}$, $3/8^{th}$) Gaussian Quadrature formula, Numerical Solution to ODE :- Euler and Runge-Kutta methods

(I) BISECTION METHOD

$y = f(x)$

$$\left.\begin{array}{l} f(a) < 0 \text{ or } > 0 \\ f(b) > 0 \text{ or } < 0 \end{array}\right\} \text{Root lies in b/w these points}$$

1) Find the root of the equation x^3-4x-9 using Bisection method correct upto 3 decimal places.

$f(x) = x^3-4x-9$

$f(0) = 0-4(0)-9 < 0$

$f(1) = 1-4(1)-9 < 0$

$f(2) = 8-8-9 < 0$

$f(3) = 27-12-9 > 0$

$x_0 = 2$

$x_1 = 3$

$x_2 = \frac{x_1+x_2}{2} = \frac{2+3}{2} = 2.5$

$f(2.5) = (2.5)^3-4(2.5)-9$

$f(2.5) = -3.375 < 0$

$\frac{2.5+3}{2} = 2.75$

$f(2.75) = (2.75)^3-4(2.75)-9$

$f(2.75) = 0.796 > 0$

$\frac{2.5+2.75}{2} = 2.625$

$f(2.625) = (2.625)^3-4(2.625)-9$

$= -1.412 < 0$

$\frac{2.75+2.625}{2} = 2.6875$

$f(2.6875) = -0.3391 < 0$

$\frac{2.6875+2.75}{2} = 2.71875$

$f(2.71875) = 0.22000 > 0$

$\frac{2.6875+2.7187}{2} = 2.7031$

$f(2.7031) = -0.0615 < 0$

$\frac{2.7187+2.7031}{2}$

$f(2.7109) = 0.0796 > 0$

$\frac{2.7031+2.7109}{2} = \boxed{2.707}$ $= 2.7109$

$f(2.707) = -0.0084 < 0$

$$\frac{2.7109 + 2.707}{2} = 2.7089$$

$f(2.7089) = 0.0426 > 0$

$$\frac{2.707 + 2.7089}{2} = 2.70795$$

$f(2.70795) = 0.02577 > 0$

$$\cancel{\frac{2.70779 + 2.70779}{2}} =$$

$$\frac{2.7070 + 2.70605}{2} = 2.70654$$

$f(2.70654) = 0.00021 > 0$

So the root of q^n is 2.706

(2) Using the bisection find the negative root of $x^3-4x+9=0$ 4

~~$f(0)=9$~~ Negative root put $x=-x$

~~$f(-1)=$~~

$$(-x)^3-4(-x)+9=0$$

~~$-x \times -x \times -x$~~

~~$-x^3$~~

$$-x^3+4x+9=0$$

Take - common

$$x^3-4x-9=0$$

~~$-x^3-4x-9$~~

$$-(x^3-4x-9)=0$$

~~$-x^3+4x-9=0$~~

$$\boxed{x^3-4x-9}$$

---

(3) Using the bisection method, find an approximate root of the eq^n $\sin x = 1/x$ that lies b/w $(1, 1.5)$ [Measured in radians]

Carry out computation upto 7th stage

$f(x)=x\sin x-1=0$

$f(x)= x\sin x-1$

$f(1)= \sin 1-1$

$= 0.8414-1 \Rightarrow -0.1586 < 0$

$f(1.5)= 1.5\sin 1.5-1$

$= 0.4962 > 0$

$\frac{1+1.5}{2} = 1.25$

(1) $f(1.25) = 1.25\sin(1.25)-1$

$= 0.1862 > 0$

$\frac{1+1.25}{2} = 1.125$

(2) $f(1.125) = 0.0150 > 0$

$\frac{1+1.125}{2} =$ 1.0625

$f(1.0625) = -0.0718 < 0$

(3) $\dfrac{1.125 + 1.0625}{2} = 1.09375$

$f(1.09375) = -0.028 < 0$

$\cancel{\dfrac{1.09375 + 1.0625}{2}}$

$\dfrac{1.09375 + 1.125}{2}$

$= 1.10937$

(4) $f(1.10937) = < 0$

$\dfrac{1.10937 + 1.125}{2}$

$= 1.1171$

(5) $f(1.1171) = > 0$

$\dfrac{1.10937 + 1.1171}{2}$

1.1132

(6) $f(1.11328) = < 0$

(3) Find the root of the eqn $\cos x = xe^{x}$ using bisection method correct upto 6th stage

$[0.5156]$

(8421), (421), (2,1)

Computer Programming

Binary Systems, Arithmatic & logical operation on numbers, Octal & Hexadecimal systems, conversion to and from decimal system, Algebra of Binary number, Elements of system and concept of Memory, Basic logic gates & Truth table. Boolean Algebra and Normal forms, Representation of unsigned & signed integers & Reals, Double precision of reals and log integers, Algorithm & Flow chart for Numerical analysis problems. Done

(1)

System	Base	Symbols	weights
Decimal	10	0 - 9	10
Binary	2	0, 1	2
Octal	8	0 - 7	8
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	16

Top to bottom for decimal left side after division
Bottom to top for right side after division

① $(170)_8$ convert into base 2, base 10 & base 16

$$170 = 0\times8^0 + 7\times8^1 + 1\times8^2$$
$$= 0 + (7\times8) + (1\times64)$$
$$= 0 + 56 + 64$$
$$(170)_8 = (120)_{10}$$

$$
\begin{array}{r|l}
2 & 120 \\
2 & 60 - 0 \\
2 & 30 - 0 \\
2 & 15 - 0 \\
2 & 7 - 1 \\
2 & 8 - 1 \\
 & 1 - 1
\end{array}
$$

$$(170)_8 = (1\,111000)_2$$

$$(170)_8 = (78)_{16} \qquad \begin{array}{r|l} 16 & 120 \\ & 7 - 8 \end{array}$$

② $(8488)_{16}$

$$8488 = 8\times16^0 + 8\times16^1 + 4\times16^2 + 8\times16^3$$
$$= 8 + 128 + 1024 + 32768$$
$$(8488)_{16} = (33928)_{10}$$

$$(8488)_{16} = (1000\,0100\,1000\,1000)_2$$

$$(8488)_{16} = (102210)_8$$

10, 11, 12, 13, 14, 15
A B C D E F

$(21EA)_{16}$

$(21EA) = \cancel{2\times16^0} + \cancel{1\times16^1} + \cancel{E\times16^2} + \cancel{A\times16^3}$

$= \cancel{2+16+3584+40960}$

$= \cancel{44562}$

$A\times16^0 + E\times16^1 + 1\times16^2 + 2\times16^3$

$= 10 + 224 + 256 + 8192$

$(21EA)_{16} = (8682)_{10}$

$(8682)_{16} = \cancel{(111110111010)}$

$\to (\underbrace{0010}\ \underbrace{0001}\ \underbrace{1110}\ \underbrace{1010})_2$

$(8682)_{16} = (20752)_8$

1001
1111
1000

Addition

$0+0=0$

$0+1=1$

$1+0=1$

$1+1=0$ with $\boxed{1}$ carry

Substraction

$0-0=0$

$1-0=1$

$0-1=1$ with borrow $\boxed{1}$

$1-1=0$

$0\times0=0$ Multiplication.

$1\times0=0$

$0\times1=0$

$1\times1=1$

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(1) $(101)_2 + (1101)_2$

$$\begin{array}{r} 101 \\ 1101 \\ \hline \boxed{1}0010 \end{array}$$

$\rightarrow \boxed{10010}$

(2) $(1001)_2 \times (1101)_2$

$$\begin{array}{r} 1001 \times 1101 \\ \hline 1001 \\ 00000 \\ 100100 \\ 1001000 \\ \hline \end{array}$$

$(1110101)_2$

③ $(111101)_2 - (10010)_2$

9866
69
9797

$$
\begin{array}{rcccccc}
 & 1 & 1 & 1 & 1 & 0 & 1 \\
(-) & 0 & 1 & 0 & 0 & 1 & 0 \\
\hline
 & (1 & 0 & 1 & 0 & 1 & 1)_2
\end{array}
$$

$$
\begin{array}{ccccc}
1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 \\
\hline
1 & 0 & 0 & 0 & 1
\end{array}
$$

④

$$
\begin{array}{cccccc}
1 & 1 & 1 & 1 & 0 & 1 \\
 & 1 & 0 & 1 & 1 & 1 \\
\hline
1 & 0 & 0 & 1 & 1 & 0
\end{array}
$$

$$
\begin{array}{cccccc}
1 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 \\
\hline
0, & 1 & 1 & 1 & 1 & 0
\end{array}
$$

$$
\begin{array}{lcccccc}
1\text{'s complement} & 0 & 0 & 0 & 0 & 1 & 0 \\
 & & 1 & 0 & 1 & 1 & 1 \\
\hline
 & & 1 & 1 & 0 & 0 & 1 \\
 & & & & & & 1 \\
\hline
 & & 1 & 1 & 0 & 0 & 0
\end{array}
$$

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