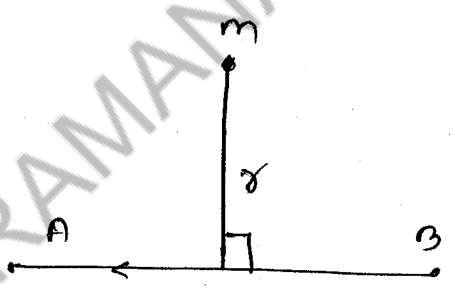
MECHANICS :- 30 - 40 Marks

Generalized co-ordinates, D'Alembert's principle & lagrange's equations, Hamilton equations ; Moment of Inertia ; Motion of rigid bodies in 2 dimensions.

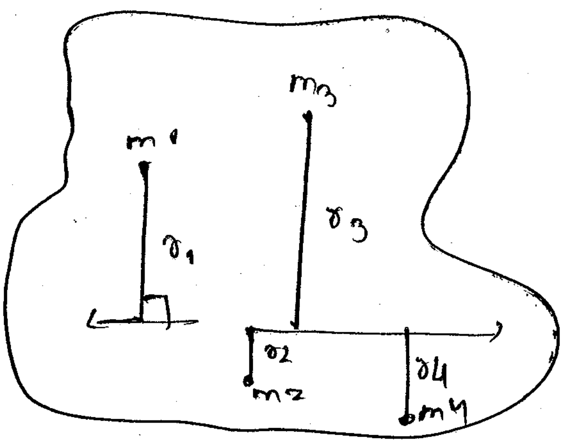
Moment of Inertia :-

Rigid body → Rigid body has the invariable size & shape & distance b/w 2 particles always remains same.

Moment of Inertia of particle :-



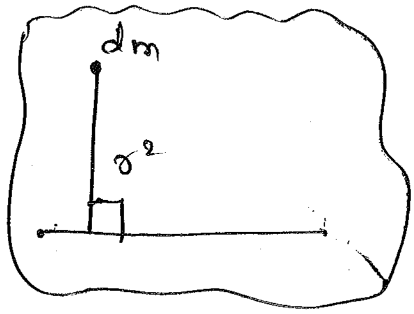
$$I = mr^2$$



$$I = \sum_{p=1}^{n} m_p r_p^2$$

M.I of continuous distribution of mass :-

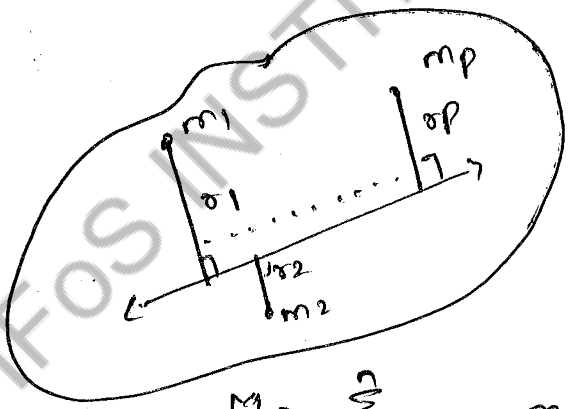
$$I = \int r^2\, dm$$



$$M = \int dm$$

Radius of gyration :-

$$M.I = \sum_{p=1}^{n} m_p r_p^2$$



$$I = M K^2$$

$$\boxed{K^2 = \frac{I}{M}}$$

$$M = \sum_{p=1}^{n} m_p = m_1 + m_2 + \dots m_n$$

K is called as Radius of Gyration.

K^2 Units = cm^2, m^2

K = cm, m...

$$K^2 = \frac{I}{M} = \frac{\int r^2 dm}{\int dm}$$

↓

Radius of gyration of continuous mass distribution

Product of Inertia :- If $(x_1, y_1), (x_2, y_2) \dots \dots (x_n, y_n)$

are the co-ordinates of the particles $m_1, m_2 \ldots m_n$ and 2 mutually perpendicular lines OX & OY. then the product of inertia with respect to the lines OX & OY is

$$P = m_1x_1y_1 + m_2x_2y_2 + \ldots m_nx_ny_n$$

$$P.I = P = \sum_{p=1}^{n} m_p x_p y_p .$$

<u>Product of Inertia (P.I) of 3 mutually $\perp$lar axis :-</u>

If 3 mutually $\perp$lar axis OX, OY, OZ in the space & $(x_1, y_1, z_1), (x_2, y_2, z_2) \ldots (x_n, y_n, z_n)$ are respective co ordinates of the particle masses $m_1, m_2 \ldots m_n$ then

P.I of the s/m with respect to OX & OY $= \sum_{p=1}^{n} m_p x_p y_p$

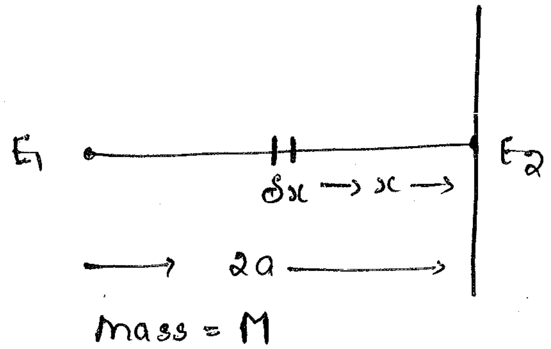
-"- OY & OZ $= \sum_{p=1}^{n} m_p y_p z_p$

-"- OX & OZ $= \sum_{p=1}^{n} m_p x_p z_p .$

M.I in some simple cases :-

Rod :-

(1) M.I of Rod length '2a' and mass M about a line through one of its extremeties ⊥lar to its length.



Mass of elementary length δx =

$$\frac{M}{2a} \times \delta x$$

M.I of the whole rod

$$\int_0^{2a} \frac{M}{2a} \delta x . x^2$$

$$= \int_0^{2a} \frac{M}{2a} x^2 \, dx$$

$$= \frac{M}{2a} \left. \frac{x^3}{3} \right|_0^{2a}$$

$$= \frac{M}{6a} \left[8a^3 - 0 \right]$$

$$\boxed{\text{M.I of rod} \Longrightarrow \frac{4Ma^2}{3}}$$

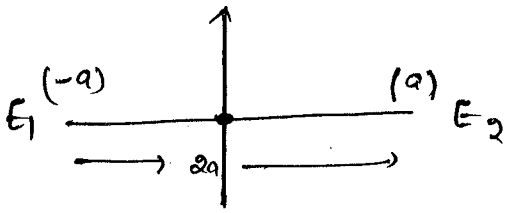
a = half of length of rod   M = mass of rod

Full length = 2a

* If m.I of rod where length of rod is 'a' & mass M is

$$\Rightarrow \frac{Ma^2}{3}$$

*



Line is ⊥^lar to midpoint of the rod.

$$= \int_{-a}^{a} \frac{M}{2a} x^2 . dx$$

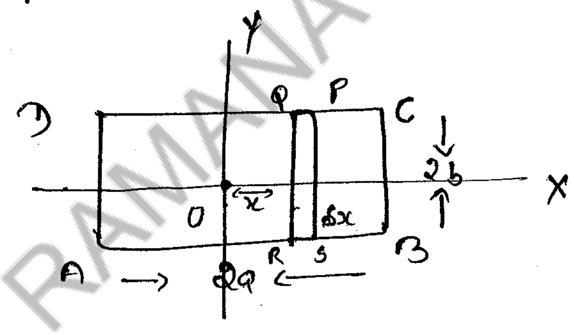
$$= 2\int_{0}^{a} \frac{M}{2a} x^2 . dx$$

$$= \frac{M}{a} \frac{x^3}{3} \Big|_0^a$$

$$\Rightarrow \frac{Ma^2}{3}$$

(2) Rectangular lamina :

M.I of Rectangular lamina about a line through its centre & parallel to one of its edges are . . . .



Mass = M

The mass of strip PQRS is

$$2b\,\delta x \frac{M}{4ab} = \frac{M}{2a}\delta x$$

M.I of the strip about OY is $\int_{-a}^{a} \frac{M}{2a} . \delta x\, x^2$

FLUID DYNAMICS :-

Equation of continuity ; Eulers eqn of motion for Inviscid flow ; Stream lines , path of a particle ; potential flow , 2-dimensional & asymmetric motion ; sources and sinks , vortex motion ; Navier - Stokes eqn for a viscous fluid.

EQUATION OF CONTINUITY :-

Material local and Convective derivatives :-

$$\frac{D}{Dt} = \frac{\delta}{\delta t} + q.\nabla$$

$$q.\nabla = q_1 \frac{\delta}{\delta x} + q_2 \frac{\delta}{\delta y} + q_3 \frac{\delta}{\delta z}$$

$\downarrow$

It is an operator

$$\nabla = i\frac{\delta}{\delta x} + j\frac{\delta}{\delta y} + k\frac{\delta}{\delta z}$$

$$\nabla f = i\frac{\delta f}{\delta x} + j\frac{\delta f}{\delta y} + k\frac{\delta f}{\delta z}$$

f be an scalar point function

$\nabla f =$ grad f

$\nabla . V =$ div V , $V = v_1 i + v_2 j + v_3 k$

$$\nabla \cdot V = \frac{\delta v_1}{\delta x} + \frac{\delta v_2}{\delta y} + \frac{\delta v_3}{\delta z}$$

$$\text{Curl} f = \nabla \times f$$

$f = f_1 i + f_2 j + f_3 k$

$$\begin{vmatrix} i & j & k \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

if curl $f = 0$ irrotational vector

$$\frac{D}{Dt} = \frac{\delta}{\delta t} + (q \cdot \nabla)$$

(i) $\frac{D}{Dt}$ is called material [particle or substantial] derivative

(ii) $\frac{\delta}{\delta t}$ is called local derivative

(iii) $q \cdot \nabla$ is called convective derivative.

$$\frac{Df}{Dt} = \frac{\delta f}{\delta t} + (q \cdot \nabla) f$$ called as eqn of continuity

<u>Eqn of continuity in cartesian coordinates</u> :-

$$\frac{D\rho}{Dt} + \rho\left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}\right) = 0$$ is called eqn of continuity in cartesian co-ordinates

It is true at all points of the fluid which is free from sources & sinks.

ρ = Density of fluid particle

ρ = function of $(x, y, z) = f(x, y, z)$

$\rho = \frac{\text{mass}}{\text{Volume}}$

u, v, w are velocity components.

$\vec{q} = ui + vj + wk$

u, v, w are some functions of x, y, z, t

<u>Note:</u>

- If the fluid is incompressible and homogenous then

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

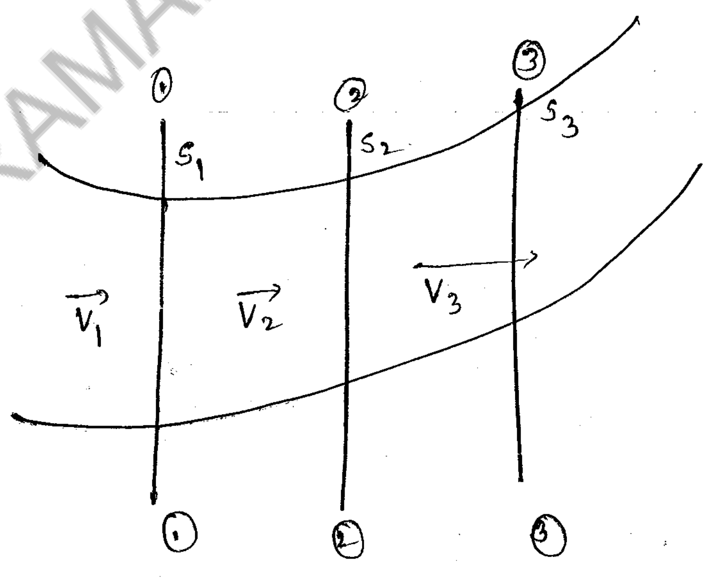
$\nabla \cdot q = 0$

$div\, q = 0$

$q = ui + vj + wk$

q is solenoidal vector.

<u>Eqn of continuity for liquid flow through a channel or pipe:-</u>



S_1, S_2, S_3 are cross sectional area, V_1, V_2 & V_3 are velocities at ①-① channel, ②-② channel & ③-③ channel

$$S_1 V_1 = S_2 V_2 = S_3 V_3$$

S.T

Q. In a 2 dimensional steady flow, incompressible field the eqn of continuity is satisfied with velocity components in rectangular co-ordinates is given by.

$$u(x,y) = \frac{K(x^2-y^2)}{(x^2+y^2)^2} ; \quad V(x,y) = \frac{2kxy}{(x^2+y^2)^2}.$$

where K is an arbitrary constant.

Sol :

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0 ,$$

$$\frac{\delta u}{\delta x} = K \frac{(2x)(x^2+y^2)^2 - (x^2-y^2)\, 2(x^2+y^2).2x}{(x^2+y^2)^4}$$

$$= K\left[\frac{2x\,(x^2+y^2)^2 - 4x\,(x^2-y^2)(x^2+y^2)}{(x^2+y^2)^4}\right]$$

$$\frac{\delta v}{\delta y} = \frac{2K\left[x\left[x^2+y^2\right]^2 - 2xy\,(x^2+y^2)\,2y\right]}{(x^2+y^2)^4}$$

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = K\left[\frac{2x\,(x^2+y^2)^2 - 4x\,(x^2-y^2)(x^2+y^2)}{(x^2+y^2)^4}\right]$$

$$+ \frac{2K\left[x\left[x^2+y^2\right]^2 - 2xy\,(x^2+y^2)\,2y\right]}{(x^2+y^2)^4}$$

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0$$

∴ The given fluid is incompressible

(5) In a 3Dimensional incompressible flow the velocity components in y & z directions are

$v = ax^3 - by^2 + cz^2$, $w = bx^3 - cy^2 + az^2x$

determine the missing component of velocity distribution such that continuity eqn is satisfied.