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Analytical Geometry

Indian Forest Service (IFoS) Maths Optional Previous Year Questions (PYQ) from 2025 to 2009

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2025

1. Find the equation of the plane passing through the points $(2, 2, 1)$ and $(9, 3, 6)$ and perpendicular to the plane $2x + 6y + 6z = 9$. **[8 Marks]**

2. (i) Find the shortest distance between the straight lines **[8+7=15 Marks]**

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{and} \quad \frac{x-5}{4} = \frac{y-4}{4} = \frac{z-5}{5}.$$

Also show that the lines are coplanar.
(ii) Show that the enveloping cylinder of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

whose generators are parallel to the line

$$\frac{x}{0} = \frac{y}{\pm\sqrt{a^2 - b^2}} = \frac{z}{c},$$

meets the plane $z = 0$ in circles.

3. Find the equation of the cone whose vertex is the point $(1, 2, 3)$ and guiding curve is **[15 Marks]**

$$x^2 + y^2 + z^2 = 4, \qquad x + y + z = 1.$$

4. Find the equations of the spheres passing through the circle **[10 Marks]**

$$x^2 + y^2 + z^2 - 5 = 0, \qquad 2x + 3y + z - 3 = 0$$

and touching the plane $3x + 4z - 15 = 0$.

2024

5. Show that if $ax^2 + 2hxy + by^2 + 2gx + 1 = 0$ represents two straight lines, then $b < 0$ and $bg^2 + h^2 = ab$. **[8 Marks]**

6. (i) Reduce the equation **[6+9=15 Marks]**

$$(c^2 + d^2)(x^2 + y^2) = (cx + dy + 2f)^2$$

to its canonical form and show that it represents a parabola. Find the latus rectum of the parabola.
(ii) A variable sphere passes through the points $(0, 0, \pm c)$ and cuts the lines

$$y - x\tan\theta = 0 = z - c \quad \text{and} \quad y + x\tan\theta = 0 = z + c$$

in the points P and Q. If $|PQ| = 2a$, where a is a positive number, then show that the centre of all such spheres lies on the circle

$$x^2 + y^2 = (a^2 - c^2)\,\mathrm{cosec}^2\, 2\theta, \qquad z = 0.$$

7. (i) Show that the equation of the plane containing the line **[8+7=15 Marks]**

$$\frac{y}{b} + \frac{z}{c} = 1, \qquad x = 0,$$

and parallel to the line

$$\frac{x}{a} - \frac{z}{c} = 1, \qquad y = 0,$$

is

$$\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0.$$

Further show that if $2d$ is the shortest distance between the given lines, then

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{d^2}.$$

(ii) A variable plane is parallel to the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

and meets the coordinate axes in A, B, C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0.$$

8. Find the equations of the generating lines of the hyperboloid **[10 Marks]**

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$$

passing through the point $(2, 3, -4)$.

2023

9. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two intersecting straight lines, then show that the square of the distance of the point of intersection from the origin is **[8 Marks]**

$$\frac{c(a+b) - (f^2 + g^2)}{ab - h^2}.$$

10. (i) Find the equation of the plane which passes through the point $(2, 1, -1)$ and is orthogonal to each of the planes $x - y + z = 1$ and $3x + 4y - 2z = 0$. **[8+7=15 Marks]**
(ii) Find the equation of the sphere for which the circle

$$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0, \qquad 2x + 3y + 4z = 8$$

is a great circle.

11. (i) Find the coordinates of the vertex, focus and the length of the latus rectum of the principal sections of the paraboloid given by **[8+7=15 Marks]**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c}.$$

(ii) Find the nature of the quadric surface given by the equation

$$2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z = 5.$$

Also find its associated characteristics, principal axes and principal planes.

12. Prove that the straight lines **[10 Marks]**

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} \quad \text{and} \quad \frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$$

intersect and find the equation of the plane containing them. Also find their point of intersection.

2022

13. A variable plane is at a constant distance of 6 units from the origin and meets the axes in $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$. Find the locus of the centroid of the triangle ABC. **[8 Marks]**

14. Obtain the coordinates of the points where the shortest distance line between the straight lines **[15 Marks]**

$$\frac{x-3}{-1} = \frac{y-2}{2} = \frac{z-2}{-1} \quad \text{and} \quad \frac{x-2}{2} = \frac{y+3}{3} = \frac{z+2}{2}$$

meets them. Also find the magnitude of the shortest distance and the equation of the shortest distance line between the straight lines mentioned above.

15. Find the equation of the cylinder whose generators intersect the curve **[15 Marks]**

$$2x^2 + 3y^2 = 4z, \qquad x - y + 2z = 3$$

and are parallel to the line $3x = -2y = 4z$.

16. Reduce the equation **[15 Marks]**

$$3x^2 + 6yz - y^2 - z^2 - 6x + 6y + 2z + 2 = 0$$

to a canonical form and mention the name of the surface it represents.

2021

17. Find the equation of the plane passing through the points $(1,-1,1)$ and $(-2,1,-1)$ and perpendicular to the plane $2x + y + z + 5 = 0$. **[8 Marks]**

18. Find the shortest distance between the line $y = 10 - 2x$ and the ellipse **[15 Marks]**

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

using Lagrange's method of multipliers.

19. Find the equation of the cone whose vertex is $(1,2,1)$ and which passes through the circle **[15 Marks]**

$$x^2 + y^2 + z^2 = 5, \qquad x + y - z = 1.$$

20. Show that the straight lines whose direction cosines are given by the equations $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are parallel if **[15 Marks]**

$$\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$$

and perpendicular if

$$a^2(v+w) + b^2(w+u) + c^2(u+v) = 0.$$

21. Find the equation of the sphere passing through the points $(1,1,2)$ and $(1,-1,2)$ and having centre on the line **[10 Marks]**

$$x + y - z - 1 = 0 = 2x + y - z - 2.$$