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Calculus

Indian Forest Service (IFoS) Maths Optional Previous Year Questions (PYQ) from 2025 to 2009

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IAS, UPSC, CIVIL SERVICES, IFoS MAINS EXAMINATION MATHEMATICS OPTIONAL STUDY MATERIALS

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2025

1. Amreek has n number of children by his first wife. Shaina has $(n+1)$ children by her first husband. They marry and have children of their own also. The whole family now has 12 children. It is assumed that children of Amreek from his first wife do not fight among themselves, and likewise, children of Shaina by her first husband do not fight among themselves. Find the maximum possible number of fights between children that can take place. **[8 Marks]**

2. Prove that $\Gamma\left(n+\frac{1}{2}\right) = \dfrac{\sqrt{\pi}\,\Gamma(2n+1)}{2^{2n}\Gamma(n+1)}$. **[8 Marks]**

3. Find the expansion of $(\sin^{-1} x)^2$ in terms of ascending powers of x. **[8 Marks]**

4. Evaluate $\lim\limits_{x\to\infty}\left(x\sqrt{x^2+1}-x^2\right)$. **[7 Marks]**

5. Prove that $\dfrac{y-x}{1+y^2} < \tan^{-1} y - \tan^{-1} x < \dfrac{y-x}{1+x^2}$, $0 < x < y$. Hence or otherwise, show that $\dfrac{\pi}{4}+\dfrac{3}{25} < \tan^{-1}\left(\dfrac{4}{3}\right) < \dfrac{\pi}{4}+\dfrac{1}{6}$. **[10 Marks]**

6. Prove that the area included between the folium $x^3 + y^3 = 3axy$ and its asymptote is equal to the area of its loop. **[15 Marks]**

2024

7. Find the relation between the radii of a right circular cylinder and a cone if the former with maximum possible curved surface area is inscribed in the latter. **[8 Marks]**

8. Find the limit of $(\cot x - \tan x)^{1/\log_e x}$, when $x \to 0$. **[8 Marks]**

9. Evaluate the volume of the solid formed by rotating the curve $r = a(1+\cos\theta)$ about the initial line. **[15 Marks]**

10. If $u = \exp\left\{\sin^{-1}\left(\dfrac{x+y}{\sqrt{x}-\sqrt{y}}\right)\right\}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{2}u\tan(\log_e u)$. **[10 Marks]**

11. How many loops are generated by the curve $r = a\sin 3\theta$? Find the sum of the areas of all the loops. **[8 Marks]**

12. Deduce the asymptote of the curve $r\log_e\theta = a$. **[7 Marks]**

2023

13. Test the convergence of the improper integral $\displaystyle\int_a^b \frac{dx}{(x-a)^n}$. **[8 Marks]**

14. If $u = z\sin\left(\dfrac{y}{x}\right)$, where $x = 3r^2 + 3s$, $y = 4r - 2s^3$, $z = 2r^2 - 3s^2$, then find $\dfrac{\partial u}{\partial r}$ and $\dfrac{\partial u}{\partial s}$. **[8 Marks]**

15. Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$ Then show that (i) $D_1 f(0,0)$ and $D_2 f(0,0)$ exist, (ii) $f(x,y)$ is continuous at $(0,0)$ by ε–δ method, and (iii) $f(x,y)$ is not differentiable at $(0,0)$. **[15 Marks]**

16. Find the maximum and minimum distances from the origin to the curve $5x^2 + 6xy + 5y^2 - 8 = 0$, using Lagrange's multiplier method. **[10 Marks]**

17. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. **[15 Marks]**

2022

18. Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right)$. **[8 Marks]**

19. If $x + y + z = u$, $y + z = uv$, $z = uvw$, then determine $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. **[8 Marks]**

20. Using Lagrange's undetermined multipliers method, find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **[15 Marks]**

21. Find the centre of mass of a solid bounded below by $x^2 + y^2 \leq 4$, $z = 0$ and above by the paraboloid $z = 4 - x^2 - y^2$. Take the density of the solid as uniform. **[15 Marks]**

22. Show that the improper integral $\int_0^\infty x^{m-1} e^{-x}\, dx$ is convergent if $m > 0$. **[10 Marks]**

2021

23. Given $\Delta(x) = \begin{vmatrix} f(x+\alpha) & f(x+2\alpha) & f(x+3\alpha) \\ f(\alpha) & f(2\alpha) & f(3\alpha) \\ f'(\alpha) & f'(2\alpha) & f'(3\alpha) \end{vmatrix}$, where f is a real-valued differentiable function and α is a constant. Find $\lim_{x \to 0} \frac{\Delta(x)}{x}$. **[10 Marks]**

24. Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$. **[10 Marks]**

25. Given that $f(x, y) = |x^2 - y^2|$, find $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$. Hence show that $f_{xy}(0, 0) = f_{yx}(0, 0)$. **[15 Marks]**

26. If $u = x^2 + y^2$, $v = x^2 - y^2$, where $x = r\cos\theta$ and $y = r\sin\theta$, then find $\frac{\partial(u, v)}{\partial(r, \theta)}$. **[7 Marks]**

27. If $\int_0^x f(t)\, dt = x + \int_x^1 f(t)\, dt$, then find the value of $f(1)$. **[5 Marks]**

28. Express $\int_a^b (x - a)^m (b - x)^n\, dx$ in terms of Beta function. **[8 Marks]**

29. Show that the entire area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $\frac{3}{8}\pi a^2$. **[15 Marks]**