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UPSC MATHS
OPTIONAL COACHING

Calculus

Previous year Questions
from 2025 to 1992

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2025

1. A rectangular sheet of metal of length 6 meters and width 2 meters is given. Four equal squares are removed from the four corners. The sides of this sheet are now folded up to form an open rectangular box. Find approximately the height of the box, such that the volume of the box is maximum. **[10 Marks]**

2. Given that $f(x+y) = f(x)f(y)$ for all real x and y, $f(x) \neq 0$ for any real x and $f'(0) = 2$. Show that for all real x, $f'(x) = 2f(x)$. Hence find $f(x)$. **[10 Marks]**

3. Using Mean Value Theorem, prove that $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$. **[15 Marks]**

4. Evaluate $\iint_R y\,dx\,dy$, where R is the region bounded by $y = x$ and $y = 4x - x^2$. **[10 Marks]**

5. If $u(x,y) = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, where f and g are arbitrary functions, then show that **[10 Marks]**

$$(i) \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = xf\left(\frac{y}{x}\right),$$

and

$$(ii) \quad x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0.$$

6. If $f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}$ when $(x,y) \neq (0,0)$ and $f(0,0) = 0$, then find $f_{xy}(0,0)$ and $f_{yx}(0,0)$. **[15 Marks]**

2024

7. Discuss the continuity of the function **[10 Marks]**

$$f(x) = \begin{cases} \dfrac{1}{1-e^{-1/x}}, & x \neq 0, \\ 0, & x = 0, \end{cases}$$

for all values of x.

8. Expand $\ln x$ in powers of $(x-1)$ by Taylor's theorem and hence find the value of $\ln(1.1)$ correct up to four decimal places. **[10 Marks]**

9. If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If yes, find the relationship. **[15 Marks]**

10. Find the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α. **[20 Marks]**

11. Using double integration, find the area lying inside the cardioid $r = a(1+\cos\theta)$ and outside the circle $r = a$. **[15 Marks]**

2023

12. Find the values of p and q for which **[10 Marks]**

$$\lim_{x\to 0}\frac{x(1+p\cos x)-q\sin x}{x^3}=1$$

exists.

13. Examine the convergence of the integral $\int_0^1 \frac{\log x}{1+x}\,dx$. **[10 Marks]**

14. Evaluate the triple integral which gives the volume of the solid enclosed between the two paraboloids $z=5(x^2+y^2)$ and $z=6-7x^2-y^2$. **[15 Marks]**

15. Justify whether $(0,0)$ is an extreme point for the function $f(x,y)=2x^4-3x^2y+y^2$. **[15 Marks]**

16. Trace the curve $y^2(x^2-1)=2x-1$. **[20 Marks]**

2022

17. Evaluate $\lim_{x\to\infty}(e^x+x)^{1/x}$. **[10 Marks]**

18. Examine the convergence of $\int_0^2 \frac{dx}{2x-x^2}$. **[10 Marks]**

19. A wire of length l is cut into two parts which are bent in the form of a square and a circle respectively. Using Lagrange's method of undetermined multipliers, find the least value of the sum of the areas so formed. **[15 Marks]**

20. Use double integration to calculate the area common to the circle $x^2+y^2=4$ and the parabola $y^2=3x$. **[15 Marks]**

2021

21. Given **[10 Marks]**

$$\Delta(x) = \begin{vmatrix} f(x+\alpha) & f(x+2\alpha) & f(x+3\alpha) \\ f(\alpha) & f(2\alpha) & f(3\alpha) \\ f'(\alpha) & f'(2\alpha) & f'(3\alpha) \end{vmatrix},$$

where f is a real-valued differentiable function and α is a constant. Find $\lim_{x\to 0} \frac{\Delta(x)}{x}$.

22. Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$. **[10 Marks]**

23. Given that $f(x,y) = |x^2 - y^2|$, find $f_{xy}(0,0)$ and $f_{yx}(0,0)$. Hence show that $f_{xy}(0,0) = f_{yx}(0,0)$. **[15 Marks]**

24. If $u = x^2 + y^2$, $v = x^2 - y^2$, where $x = r\cos\theta$ and $y = r\sin\theta$, then find $\frac{\partial(u,v)}{\partial(r,\theta)}$. **[7 Marks]**

25. If $\int_0^x f(t)\,dt = x + \int_x^1 f(t)\,dt$, then find the value of $f(1)$. **[5 Marks]**

26. Express $\int_a^b (x-a)^m (b-x)^n\,dx$ in terms of Beta function. **[8 Marks]**

27. Show that the entire area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $\frac{3\pi a^2}{8}$. **[15 Marks]**

2020

28. Evaluate $\lim_{x\to\frac{\pi}{4}}(\tan x)^{\tan 2x}$. **[10 Marks]**

29. Find all the asymptotes of the curve $(2x+3)y=(x-1)^2$ **[10 Marks]**

30. Evaluate $\int_0^1 \tan^{-1}\left(1-\frac{1}{x}\right)dx.$ **[15 Marks]**

31. Consider the function $f(x)=\int_0^x (t^2-5t+4)(t^2-5t+6)dt$

(i) Find the critical points of the function $f(x)$

(ii) Find the points at which local minimum occurs.

(iii) Find the points at which local maximum occurs.

(iv) Find the number of zeros of the function $f(x)$ in $[0,5]$ **[20 Marks]**

32. Find an extreme value of the function $u=x^2+y^2+z^2$ subject to the condition $2x+3y+5z=30$ by using Lagrange's method of undetermined multiplier. **[20 Marks]**

2019

33. Let $f:\left[0,\frac{\pi}{2}\right]\to R$ be a continuous function such that $f(x)=\frac{\cos^2 x}{4x^2-\pi^2}$, $0\le x\le\frac{\pi}{2}$. Find the value of $f\left(\frac{\pi}{2}\right)$ **[10 Marks]**

34. Let $f:D(\subseteq R^2)\to R$ be a function and $(a,b)\in D$. If $f(x,y)$ is continuous at (a,b), then show the functions $f(x,b)$ and $f(a,y)$ are continuous at $x=a$ and at $y=b$ respectively. **[10 Marks]**

35. Is $f(x)=|\cos x|+|\sin x|$ differentiable at $x=\frac{\pi}{2}$? If yes, then find its derivative at $x=\frac{\pi}{2}$ $x=\frac{\pi}{2}$ If no, then a proof of it. **[15 Marks]**

36. Find the maximum and the minimum value of the function $f(x)=2x^3-9x^2+12x+6$ on the interval $[2,3]$ **[10 Marks]**

37. If $u=\sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$ then show that $\sin^2 u$ is a homogeneous function of x and y of degree $-\frac{1}{6}$ hence show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{12}\right)$ **[12 Marks]**

38. Using the Jacobian method, show that if $f'(x)=\frac{1}{1+x^2}$ and $f(0)=0$ then $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$ **[8 Marks]**

2018

39. Determine if $\lim_{z\to 1}(1-z)\tan\frac{\pi z}{2}$ exists or not. If the limit exists, then find its value. **[10 Marks]**

40. Find the limit $\lim_{n\to\infty}\frac{1}{n^2}\sum_{r=0}^{n-1}\sqrt{n^2-r^2}$. **[10 Marks]**

41. Find the shortest distance from the point $(1,0)$ to the parabola $y^2=4x$ **[13 Marks]**

42. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolves about the x-axis. Find the volume of the solid of revolution. **[13 Marks]**

43. Let $f(x,y)=\begin{cases}xy^2, & y>0\\ -xy^2, & y\le 0\end{cases}$. Determine which of $\frac{\partial f}{\partial x}(0,1), \frac{\partial f}{\partial y}(0,1)$ and exists and which does not exist. **[12 Marks]**

44. Find the maximum and the minimum values of x^4-5x^2+4 on the interval $[2,3]$. **[13 Marks]**

45. Evaluate the integral $\int_0^a\int_{x/a}^{x}\frac{xdydx}{x^2+y^2}$ **[12 Marks]**

2017

46. Integrate the function $f(x,y)=xy(x^2+y^2)$ over the domain $R:\{-3\le x^2-y^2\le 3,\ 1\le xy\le 4\}$ **[10 Marks]**

47. Find the volume of the solid above the xy-plane and directly below the portion of the elliptic paraboloid $x^2+\frac{y^2}{4}=z$ which is cut off by the plane $z=9$ **[15 Marks]**

48. If $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

calculate $\frac{\partial^2 f}{\partial x\partial y}$ and $\frac{\partial^2 f}{\partial y\partial x}$ at $(0,0)$. **[15 Marks]**

49. Examine if the improper integral $\int_0^3\frac{2xdx}{(1-x^2)^{2/3}}$, exists. **[10 Marks]**

50. Prove that $\frac{\pi}{3}\le\iint_D\frac{dxdy}{\sqrt{x^2+(y-2)^2}}\le\pi$ where D is the unit disc. **[10 Marks]**

2016

51. Evaluate: $I=\int_0^1\sqrt[3]{x\log\left(\frac{1}{x}\right)}\,dx$ **[10 marks]**

52. Find the matrix and minimum values of $x^2+y^2+z^2$ subject to the conditions $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$ and $x+y-z=0$ **[20 marks]**

53. Let $f(x,y)=\begin{cases}\frac{2x^4-5x^2y^2+y^5}{(x^2+y^2)^2}, & x,y)\neq(0,0)\\ 0, & x,y)=(0,0)\end{cases}$ find a $\delta>0$ such that $|f(x,y)-f(0,0)|<0.01$ whenever $\sqrt{x^2+y^2}<\delta$ **[15 marks]**

54. Find the surface area of the plane $x+2y+2z=12$ cut off by $x=0, y=0$ and $x^2+y^2=16$ **[15 marks]**

55. Evaluate $\iint_R f(x,y)dxdy$, over the rectangle $R=[0,1;0,1]$ where $f(x,y)=\begin{cases} x+y, & \text{if } x^2<y<2x^2 \\ 0, & \text{elsewhere} \end{cases}$ **[15 marks]**

2015

56. Evaluate the following limit $\lim_{x\to a}\left(2-\frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ **[10 Marks]**

57. Evaluate the following integral: $\int_{\pi/6}^{\pi/2}\frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx$ **[10 Marks]**

58. A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base. **[13 Marks]**

59. Which point of the sphere $x^2+y^2+z^2=1$ is at the maximum distance from the point $(2,1,3)$ **[13 Marks]**

60. Evaluate the integral $\iint_R (x-y)^2\cos^2(x+y)\,dxdy$ where R is the rhombus with successive vertices as $(\pi,0),(2\pi,\pi),(\pi,2\pi),(0,\pi)$ **[12 Marks]**

61. Evaluate $\iint_R \sqrt{\left|y-x^2\right|}\,dxdy$ where $R=[-1,1;0,2]$ **[13 Marks]**

62. For the function $f(x,y)=\begin{cases} \frac{x^2-x\sqrt{y}}{x^2+y}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ Examine the continuity and differentiability. **[12 Marks]**

2014

63. Prove that between two real roots $e^x\cos x+1=0$, a real root of $e^x\sin x+1=0$ lies. **[10 Marks]**

64. Evaluate: $\int_0^1\frac{\log_e(1+x)}{1+x^2}dx$. **[10 Marks]**

65. By using the transformation $x+y=u, y=uv$ evaluate the integral $\iint\{xy(1-x-y)\}^{\frac{1}{2}}\,dxdy$ taken over the area enclosed by the straight lines $x=0$, $y=0$ and $x+y=1$. **[15 Marks]**

66. Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a. **[15 Marks]**

67. Find the maximum or minimum values of $x^2+y^2+z^2$ subject to the condition $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$ interpret result geometrically **[20 Marks]**

2013

68. Evaluate $\int_0^1\left(2x\sin\frac{1}{x}-\cos\frac{1}{x}dx\right)$ **[10 Marks]**

69. Using Lagrange's multiplier method find the shortest distance between the line $y=10-2x$ and the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ **[20 Marks]**

70. Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function $f(x,y)=\begin{cases}\dfrac{xy^3}{x+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

Also distance the continuity of f_{xy} and f_{yx} at $(0,0)$. **[15 Marks]**

71. Evaluate $\iint_D xydA$ where D is the region bounded by the line $y=x-1$ and the parabola $y^2=2x+6$. **[15 Marks]**

2012

72. Define a function f of two real variables in the plane by $f(x,y)=\begin{cases}\dfrac{x^3\cos\frac{1}{y}+y^3\cos\frac{1}{x}}{x^2+y^2} & \text{for } x,y\neq 0\\ 0, \text{otherewis} & \end{cases}$

Check the continuity and differentiability of f at $(0,0)$. **[12 Marks]**

73. Let p and q be positive real numbers such that $\frac{1}{p}+\frac{1}{q}=1$ show that for real numbers $ab\geq 0$

$ab\frac{a^p}{p}+\frac{b^q}{q}$. **[12 Marks]**

74. Find the point of local extreme and saddle points of the function f for two variables defined by $f(x,y)=x^3+y^3-63(x+y)+12xy$ **[20 Marks]**

75. Defined a sequence S_n of real numbers by $S_n=\sum_{i=1}^{n}\frac{(\log(n+i)-\log n)^2}{n+1}$ does $\lim_{n\to\infty} S_n$ exist? If so compute the value of this limit and justify your answer **[20 Marks]**

76. Find all the real values of p and q so that the integral $\int_0^1 x^p\left(\log\frac{1}{x}\right)^q dx$ converges **[20 Marks]**

2011

77. Find $\lim_{(x,y)\to(0,0)}\frac{x^2y}{x^3+y^3}$ if it exists **[10 Marks]**

78. Let f be a function defined on $\mathbb{R}$ such that $f(0)=-3$ and $f'(x)\leq 5$ for all values of x in $\mathbb{R}$ How large can $f(2)$ possibly be? **[10 Marks]**

79. Evaluate:

(i) $\lim_{x\to 2} f(x)$ Where $f(x)=\begin{cases}\dfrac{x^2-4}{x-2}, & x\neq 2\\ \pi, & x=2\end{cases}$ (ii) $\int_0^1 \ell nx\,dx$. **[20 Marks]**

2010

80. A twice differentiable function $f(x)$ is such that $f(a)=0=f(b)$ and $f(c)>0$ for $a<c<b$ prove that there be is at least one point $\xi, a<\xi<b$ for which $f''(\xi)<0$ **[12 Marks]**

81. Dose the integral $\int_{-1}^{1}\sqrt{\frac{1+x}{1-x}}$ exist if so, find its value **[12 Marks]**

82. Show that a box (rectangular parallelepiped) of maximum volume V with prescribed surface area is a cube. **[20 Marks]**

83. Let D be the region determine by the inequalities $x>0, y>0, z<8$ and $z>x^2+y^2$ compute $\iiint_D 2xdxdydz$. **[20 Marks]**

84. If $f(x,y)$ is a homogeneous function of degree n in x and y, and has continuous first and second order partial derivatives then show that

$(i)\ x\frac{\partial^2 f}{\partial x}+y\frac{\partial^2 f}{\partial y}=nf$ $(ii)\ x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial\times\partial y}+y^2\frac{\partial^2 f}{\partial y^2}=n(n-1)f$ **[20 Marks]**

2009

85. Suppose the f' is continuous on $[1,2]$ and that f has three zeroes in the interval $(1,2)$ show that f'' has least one zero in the interval $(1,2)$. **[12 Marks]**

86. If f is the derivative of same function defined on $[a,b]$ prove that there exists a number $\eta\in[a,b]$ such that $\int_a^b f(t)dt=f(\eta)(b-a)$ **[12 Marks]**

87. If $x=3\pm0.01$ and $y=4\pm0.01$ with approximately what accuracy can you calculate the polar coordinate r and θ of the point $P(x,y)$ Express you estimates as percentage changes of the value that r and θ have at the point $(3,4)$ **[20 Marks]**

88. A space probe in the shape of the ellipsoid $4x^2+y^2+4z^2=16$ enters the earth atmosphere and its surface beings to heat. After one hour, the temperature at the point (x,y,z) on the probe surface is given by $T(x,y,z)=8x^2+4yz-16z+1600$ Find the hottest point on the probe surface. **[20 Marks]**

89. Evaluate $I=\iint_s xdydz+dzdx+xz^2dxdy$ where S is the outer side of the part of the sphere $x^2+y^2+z^2=1$ in the first octant. **[20 Marks]**

2008

90. Find the value of $\lim_{x\to1}\ln(1-x)\cot\frac{\pi x}{2}$. **[12 Marks]**

91. Evaluate $\int_0^1(x\ln x)^3dx$. **[12 Marks]**

92. Determine the maximum and minimum distances of the origin from the curve given by the equation $3x^2+4xy+6y^2=140$. **[20 Marks]**

93. Evaluate the double integral $\int_y^a\frac{xdxdy}{x^2+y^2}$ by changing the order of integration **[20 Marks]**

94. Obtain the volume bounded by the elliptic paraboloid given by the equations $z - x^2 + 9y^2$ & $z = 18 - x^2 - 9y^2$ **[20 Marks]**

2007

95. Let $f(x), (x \in (-\pi, \pi))$ be defined by $f(x) = \sin|x|$ is f continuous on $(-\pi, \pi)$ if it is continuous then is it differentiable on $(-\pi, \pi)$? **[12 Marks]**

96. A figure bounded by one arch of a cycloid $x = a(t - \sin t), y = a(1 - \cos t), t \in [0, 2\pi]$ and the x-axis is revolved about the x-axis. Find the volume of the solid of revolution **[12 Marks]**

97. Fin a rectangular parallelepiped of greatest volume for a give total surface area S using Lagrange's method of multipliers **[20 Marks]**

98. Prove that if $z = \phi(y + ax) + \psi(y - ax)$ then $a^2 \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x^2} = 0$ for any twice differentiable φ and ψ is a constant. **[15 Marks]**

99. Show that $e^{-x}x^n$ is bounded on $[0, \infty)$ for all positive integral values of n. Using this result show that $\int_0^\infty e^{-x}x^n dx$ exists. **[25 Marks]**

2006

100. Find a and b so that $f'(2)$ exists where $f(x) = \begin{cases} \frac{1}{|x|}, & \text{if } |x| > 2 \\ a + bx^2 & \text{if } |x| \le 2 \end{cases}$ **[12 Marks]**

101. Express $\int_0^1 x^m (1 - x^n)^p dx$ in terms of Gamma function and hence evaluate the integral $\int_0^1 x^6 \sqrt{(1 - x^2)} dx$ **[12 Marks]**

102. Find the values of a and b such that $\lim_{x \to 0} \frac{a \sin^2 x \times b \log \cos x}{x^4} = \frac{1}{2}$. **[15 Marks]**

103. If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$. **[15 Marks]**

104. Change the order of integration in $\int_x^\infty \frac{e^{-y}}{y} dy dx$ and hence evaluate it. **[15 Marks]**

105. Find the volume of the uniform ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **[15 Marks]**

2005

106. Show that the function given below is not continuous at the origin $f(x, y) = \begin{cases} 0 \text{ if } xy = 0 \\ 1 \text{ if } xy \neq 0 \end{cases}$ **[12 Marks]**

107. Let $R^2 \to R$ be defined as $f(x, y) = \frac{xy}{\sqrt{(x^2 + y^2)}}, (x, y) \neq (0, 0), f(0, 0) = 0$ prove that f_x and f_y exist at $(0, 0)$ but f is not differentiable at $(0, 0)$. **[12 Marks]**

108. If $u=x+y+z, uv=y+z$ and $uvw=z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ **[15 Marks]**

109. Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$ in terms of Beta function. **[15 Marks]**

110. Evaluate $\iiint_V zdV$ where V the volume is bounded below by the cone $x^2+y^2=z^2$ and above by the sphere $x^2+y^2+z^2=1$ lying on the positive side of the y-axis. **[15 Marks]**

111. Find the x-coordinate of the center of gravity of the solid lying inside the cylinder $x^2+y^2=2ax$ between the plane $z=0$ and the paraboloid $x^2+y^2=az$. **[15 Marks]**

2004

112. Prove that the function f defined on $[0,4]$ $f(x)=[x]$ greatest integer $\le x, x\in[0,4]$ is integrable on $[0,4]$ and that $\int_0^4 f(x)dx=6$. **[12 Marks]**

113. Shaw that $x-\frac{x^2}{2}<\log(1+x)<x-\frac{x^2}{2(1+x)}\ x>0$.

[12 Marks]

114. Let the roots of the equation in $\lambda(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ be u,v,w proving that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=-2\frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$. **[15 Marks]**

115. Prove that an equation of the form $x^n=\alpha$ where $n\in N$ and $\alpha>0$ is a real number has a positive root.

[15 Marks]

116. Prove that $\int\frac{x^2+y^2}{p}dx=\frac{\pi ab}{4}[4+(a^2+b^2)(a^{-2}+b^{-2})]$ when the integral is taken round the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and p is three length of three perpendicular from the center to the tangent. **[15 Marks]**

117. If the function f is defined by $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ then show that possesses both the partial derivative at but it is not continuous thereat. **[15 Marks]**

2003

118. Let f be a real function defined as follow:
$f(x)=x, -\le x<1$
$f(x+2)=x, \forall x\in R$
Show that f is discontinuous at every odd integer. **[12 Marks]**

119. For all real numbers x, $f(x)$ is given as $f(x)=\begin{cases}e^x+a\sin x, & x<0\\ b(x-1)^2+x-2, & x\ge 0\end{cases}$. Find values of a and b for which is differentiable at $x=0$. **[12 Marks]**

120. A rectangular box open at the top is to have a volume of $4m^3$. Using Lagrange's method of multipliers find the dimension of the box so that the material of a given type required to construct it may be least. **[15 Marks]**

121. Test the convergent of the integrals(i) $\int_0^1 \frac{dx}{x^{\frac{1}{3}}\left(1+x^2\right)}$ (ii) $\int_0^\infty \frac{\sin^2 x}{x^2}dx$ **[15 Marks]**

122. Evaluate the integral $\int_0^a \int_{\frac{y^2}{a}}^{y} \frac{ydxdy}{(a-x)\sqrt{ax-y^2}}$ **[15 Marks]**

123. Find the volume generated by revolving by the real bounded by the curves $(x^2+4a^2)y=8a^3$, $2y=x$ and $x=0$ about the y-axis. **[15 Marks]**

2002

124. Show that $\frac{b-a}{\sqrt{1-a^2}} \le \sin^{-1} b - \sin^{-1} a \le \frac{b-a}{\sqrt{1-b^2}}$ for $0<a<b<1$. **[12 Marks]**

125. Show that $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy = \frac{\pi}{4}$ **[12 Marks]**

126. Let $f(x)=\begin{cases} x^p \sin\frac{1}{x}, x\neq 0 \\ 0 \qquad x=0 \end{cases}$. Obtain condition on p such that (i)f is continuous at $x=0$ and (ii) f is differentiable at $x=0$ **[15 Marks]**

127. Consider the set of triangles having a given base and a given vertex angle show that the triangle having the maximum area will be isosceles **[15 Marks]**

128. If the roots of the equation $(\lambda-u)^3+(\lambda-v)^3+(\lambda-w)^3=0$ in λ are x,y,z. show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = -\frac{2(u-v)(v-w)(w-u)}{(x-y)(y-z)(z-x)}$. **[15 Marks]**

129. Find the center of gravity of the region bounded by the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$ and both axes I the first quadrant the density being $\rho = kxy$ where k is constant. **[15 Marks]**

2001

130. Let be defined on by setting $f(x)=x$ if x is rational and $f(x)=1-x$ if x is irrational show that is continuous at $x=\frac{1}{2}$ but is discontinuous at every other point. **[12Marks]**

131. Test the convergence of $\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}}dx$. **[12 Marks]**

132. Find the equation of the cubic curve which has the same asymptotes as $2x(y-3)^2=3y(x-1)^2$ and which touches the axis at the origin and passes though the point $(1,1)$. **[15 Marks]**

133. Find the maximum and minimum radii vectors of the section of the surface $(x^2+y^2+z^2)=a^2x^2+b^2y^2+c^2z^2$ by the plane $lx+my+nz=0$ **[15 Marks]**

134. Evaluate $\iiint (x+y+z+1)^2 dxdydz$ over the region defined by $x \geq 0, y \geq 0, z \geq 0, x+y+z \leq 1$

[15 Marks]

135. Find the volume of the solid generated by revolving the cardioid $r = a(1-\cos\theta)$ about the initial line

[15 Marks]

2000

136. Use the mean value theorem to prove that $\frac{2}{7} < \log 1.4 < \frac{2}{5}$.

[12 Marks]

137. Show that $\iint x^{2l-1} y^{2m-1} dxdy = \frac{1}{4} r^{2(l+m)} \frac{\Gamma l \, \Gamma m}{\Gamma(l+m+1)}$ for all positive values of and laying the circle $x^2 + y^2 = r^2$.

[12 Marks]

138. Find the center of gravity of the positive octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ if the density varies as xyz

[15 Marks]

139. Let $f(x) = \begin{cases} 2, & x \text{ is irrational} \\ 1, & x \text{ is rational} \end{cases}$ show that if is not Riemann integrable on $[a,b]$ **[15 Marks**

140. Show that $\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = (-1)^n \frac{n!}{x^{n+1}}\left(\log x - 1 - \frac{1}{2} - \frac{1}{3} .... - \frac{1}{n}\right)$ **[15 Marks]**

141. Find constant a and b for which $F(a,b) = \int_0^{\pi} \{\log x - ax^2 + bx^2\} dx$ is a minimum **[15 Marks]**

1999

142. Determine the set of all points where the function $f(x) = \frac{x}{1+|x|}$ is differentiable. **[20 Marks]**

143. Find three asymptotes of the curve $x^3 + 2x^2y - 4xy^2 - 8y^3 - 4x + 8y - 10 = 0$. Also find the intercept of one asymptote between the other two. **[20 Marks]**

144. Find the dimensions of a right circular cone of minimum volume which can be circumscribed about a sphere of radius a. **[20 Marks]**

145. If f is Riemann integral over every interval of finite length and $f(x+y) = f(x) + f(y)$ for every pair of real numbers x and y show that $f(x) = cx$ where $c = f(1)$ **[20 Marks]**

146. Show that the area bounded by cissoids $x = a\sin^2 t, y = a\frac{\sin^3 t}{\cos t}$ and its asymptote is $\frac{3\pi a^2}{4}$

[20 Marks]

147. Show that $\iint x^{m-1} y^{n-1}$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^m b^n}{4} \frac{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{m}{2} + \frac{n}{2} + 1\right)}$

[20 Marks]

1998

148. Find the asymptotes of the curve $(2x-3y+1)^2(x+y)-8x+2y-9=0$ and show that they intersect the curve again in their points which lie on a straight line. **[20 Marks]**

149. A thin closed rectangular box is to have one edge n times the length of another edge and the volume of the box is given to be v. Prove that the least surface s is given by $ns^3=54(n+1)^2v^2$ **[20 Marks]**

150. If $x+y=1$, Prove that $\frac{d^n}{dx^n}(x^n y^n)=n!\left[y^n-\left(\frac{n}{1}\right)^2 y^{n-1}x+\left(\frac{n}{2}\right)^2 y^{n-2}x^2+\ldots+(-1)^n x^n\right]$ **[20 Marks]**

151. Show that $\int_0^\infty \frac{x^{p-1}}{(1+x)^{p+q}}dx=B(p,q)$ **[20 Marks]**

152. Show that $\iiint \frac{dxdydz}{\sqrt{(1-x^2-y^2-z^2)}}=\frac{\pi^2}{8}$ Integral being extended over all positive values of x,y,z for which the expression is real. **[20 Marks]**

153. The ellipse $b^2x^2+a^2y^2=a^2b^2$ is divided into two parts by the line $x=\frac{1}{2}a,$ and the smaller part is rotated through for right angles about this line. Prove that the volume generated is $\pi a^2 b\left\{\frac{3\sqrt{3}}{4}-\frac{\pi}{3}\right\}$ **[20 Marks]**

1997

154. Suppose $f(x)=17x^{12}-124x^9+16x^3-129x^2+x-1$ determine $\frac{d}{dx}(f^{-1})$ if $x=-1$ it exists. **[20 Marks]**

155. Prove that the volume of the greatest parallelepiped that can be inscribe in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1\frac{8abc}{3\sqrt{3}}$ **[20 Marks]**

156. Show that the asymptotes of the cut the curve $(x^2-y^2)(y^2-4x^2)+6x^3-5x^2y-3xy^2+zy^3-x^2+3xy-1=0$ again in eight points which lie on a circle of radius 1. **[20 Marks]**

157. An area bounded by a quadrant of a circle of radius a and the tangent at its extremities revolve about one of the tangents. Find the volume so generated. **[20 Marks]**

158. Show how the changes of order in the integral $\int_0^\infty\int_0^\infty e^{-xy}\sin x\ dxdy$ leads to the evaluation of $\int_0^\infty \frac{\sin x}{x}dx$ hence evaluate it. **[20 Marks]**

159. Show that in $\lceil n\left|\left(n+\frac{1}{2}\right)\right.=\frac{\lceil\pi}{2^{2n-1}}\lceil 2n$ where $n>0$ and $\lceil n$ denote gamma function. **[20 Marks]**

1996

160. Find the asymptotes of all curves $4(x^4+y^4)-17x^2y^2-4x(4y^2-x^2)+2(x^2-2)=0$ and show that they pass thought the point of intersection of the curve with the ellipse $x^2+4y^2=4$. **[20 Marks]**

161. Show that any continuous function defined for all real x and satisfying the equation $f(x)=f(2x+1)$ for all x must be a constant function. **[20 Marks]**

162. Show that the maximum and minimum of the radii vectors of the section of the surface $(x^2+y^2+z^2)^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$ by the plane $\lambda x+\mu y+\upsilon z=0$ are given by the equation

$$\frac{a^2\lambda^2}{1-a^2r^2}+\frac{b^2\mu^2}{1-b^2r^2}+\frac{a^2\nu^2}{1-c^2r^2}=0.$$ **[20 Marks]**

163. If $u=f\left(\frac{x}{a},\frac{y}{b},\frac{z}{c}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$ **[20 Marks]**

164. Evaluate $\int_0^\infty\int_0^\infty \frac{e^{-y}}{y}dxdy$. **[20 Marks]**

165. The area cut off from the parabola $y^2=4ax$ by chord joining the vertex to an end of the latus rectum is rotated though four right angles about the chord. Find the volume of the solid so formed. **[20 Marks]**

1995

166. If g is the inverse of f and $f'(x)=\frac{1}{1+x^3}$ prove that $g(x)=1+[g(x)]^3$ **[20 Marks]**

167. Taking the nth derivative of $(x^n)^2$ in two different ways show that $1+\frac{n^2}{1^2}+\frac{n^2}{1^2.2^2}+\frac{n^2(n-1)^2}{1^2.2^2.3^2}+\ldots$ to

$(n+1)term=\frac{(2n)!}{(n!)^2}$ **[20 Marks]**

168. Let $f(x,y)$ which possesses continuous partial derivatives of second order be a homogeneous function of x and y off degree n prove that $x^2f_{xx}+2xyf_{xy}+y^2f_{yy}=n(n-1)f$. **[20 Marks]**

169. Find the area bounded by the curve $\left(\frac{x^2}{4}+\frac{y^2}{9}\right)=\frac{x^2}{4}-\frac{y^2}{9}$. **[20 Marks]**

170. Let $f(x),\ x\geq 1$ be such that the area bounded by the curve $y=f(x)$ and the lines $x=1,x=b$ is equal to $\sqrt{1+b^2}-\sqrt{2}$ for all $b\geq 1$. Does f attain its minimum? If so, what is its values? **[20 Marks]**

171. Show that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)\ldots\Gamma\left(\frac{n-1}{n}\right)=\frac{(2\pi)}{\sqrt{n}}\frac{n-1}{2}$. **[20 Marks]**

1994

172. $f(x)$ Is defined as follows: $f(x)=\begin{vmatrix}\frac{1}{2}(b^2-a^2) & \text{if } 0<x\leq a\\ \frac{1}{2}b^2-\frac{x^2}{6}-\frac{a^2}{3x} & \text{if } a<x\leq b\\ \frac{1}{3}\left(\frac{b^3-a^3}{x}\right) & \text{if } x>b\end{vmatrix}$. Prove that $f(x)$ and $f'(x)$ are

continuous but $f''(x)$ is discontinuous. **[20 Marks]**

173. If α and β lie between the least and greatest values of a,b,c prove that

$$\begin{vmatrix} f(a) & f(b) & f(c) \\ \phi(a) & \phi(b) & \phi(c) \\ \psi(x) & \psi(b) & \psi(c) \end{vmatrix} = K \begin{vmatrix} f(a) & f'(\alpha) & f(\beta) \\ \phi(a) & \phi'(\alpha) & \phi(\beta) \\ \psi(x) & \psi'(\alpha) & \psi(\beta) \end{vmatrix} \text{ where } K = \frac{1}{2}(b-c)(c-a)(a-b)$$ **[20 Marks]**

174. Prove that all rectangular parallelepipeds of same volume, the cube has the least surface **[20 Marks]**

175. Show that means of beta function that $\int_t^z \frac{dx}{(z-x)^{1-\alpha}(x-t)^{\alpha}} = \frac{\pi}{\sin \pi\alpha}(0<\alpha<1)$. **[20 Marks]**

176. Prove that the value of $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ taken over the volume bounded by the co-ordinate planes and the plane $x+y+z=1$ is $\frac{1}{2}\left(\log 2 - \frac{5}{8}\right)$. **[20 Marks]**

177. The sphere $x^2+y^2+z^2=a^2$ is pierced by the cylinder $(x^2+y^2)^2 = a^2(x^2-y^2)$ prove by the cylinder $(x^2+y^2)^2 = a^2(x^2-y^2)$ is $\frac{8a^3}{3}\left[\frac{\pi}{4} + \frac{5}{3} = \frac{4\sqrt{2}}{3}\right]$ **[20 Marks]**

1993

178. Prove that $f(x) = x^2 \sin\frac{1}{x}, x \neq 0$ and $f(x) = 0x = 0$ for is continuous and differentiable at $x=0$ but its derivative is not continuous there. **[20 Marks]**

179. If $f(x), \phi(x), \psi(x)$ have derivative when $a \le x \le b$ show that there is a value c of x lying between a and b such that $\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f(c) & \phi(c) & \psi(c) \end{vmatrix} = 0$ **[20 Marks]**

180. Find the triangle of maximum area which can be inscribed in a circle **[20 Marks]**

181. Prove that $\int_0^{\infty} e^{-ax^2}dx = \frac{\sqrt{\pi}}{2\sqrt{a}}(a>0)$ deduce that $\int_0^{\infty} x^{2n}e^{-x^2}dx = \frac{\sqrt{\pi}}{2^{n+1}}[1.3.5...(2n-1)]$ **[20 Marks]**

182. Defined Gamma function and prove that $\lceil n \left\lceil \left(n+\frac{1}{2}\right) \right\rceil = \frac{\lceil \pi}{2^{2n-1}} \lceil 2n$ **[20 Marks]**

183. Show that volume common to the sphere $x^2+y^2+z^2=a^2$ and the cylinder $x^2+y^2=ax$ is $\frac{2a^2}{9}(3\pi-4)$. **[20 Marks]**

1992

184. If $y = e^{ax}\cos bx$ prove that $y_2 - 2ay_1 + (a^2+b^2)y = 0$ and hence expand $e^{2x}\cos bx$ in powers of x Deduce the expansion of e^{ax} and $\cos bx$. **[20 Marks]**

185. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$ then prove that $dx^2+dy^2+dz^2 = dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$. **[20 Marks]**

186. Find the dimension of the rectangular parallelepiped inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ that has greatest volume. **[20 Marks]**

187. Prove that the volume enclosed by the cylinders $x^2+y^2=2ax, z^2=2$ axis $\frac{128a^3}{15}$ **[20 Marks]**

188. Find the centre of gravity of the volume formed by revolving the area bounded by the parabolas $y^2=4ax$ and $x^2=4by$ about the x-axis **[20 Marks]**

189. Evaluate the following integral in terms of Gamma function $\int_{-1}^{1}(1+x)^p(1-x)^q\,dx, \quad [p>-1, q>-1]$ and prove that $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)=\frac{2}{\sqrt{3}}\pi$ **[20 Marks]**