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Complex Analysis

IFoS (IFS) Previous Year Questions (PYQ) from 2025 to 2009

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2025

1. Find the image of $|z| < 1$ under the bilinear transformation which maps $z = 1,\ i,\ -1$ onto $w = i,\ 0,\ -i$ respectively. **[8 Marks]**

2. Evaluate the integral $\int_0^\infty \sin(x^2)\,dx$ using the method of contour integration. **[15 Marks]**

3. Let $f(z) = \dfrac{z}{(z-1)(z+2)}$. Find the Laurent series expansion of $f(z)$ in the following regions: (i) $1 < |z| < 2$ and (ii) $|z| > 2$. **[8+7 Marks]**

2024

4. Let $f(z) = u(r,\theta) + iv(r,\theta)$ be an analytic function. If $u = -r^3 \sin 3\theta$, then construct the corresponding analytic function $f(z)$ in terms of z. **[8 Marks]**

5. Determine the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2(z-2)}$ and the residue at each pole and hence evaluate $\oint_C f(z)\,dz$, where C is the circle $|z| = 2.5$. **[15 Marks]**

6. (i) Find the image of $|z - 3i| = 3$ under the mapping $w = \dfrac{1}{z}$. (ii) Find the value of the integral $\int_0^{1+i} (x - y + ix^2)\,dz$ along the straight line from $z = 0$ to $z = 1 + i$. **[8+7 Marks]**

2023

7. Find a bilinear transformation which maps the points $z = 0,\ -i,\ -1$ into $w = i,\ 1,\ 0$ respectively. **[8 Marks]**

8. (i) Let $f(z) = \ln(1+z)$. Expand $f(z)$ in a Taylor series about $z = 0$. Determine the region of convergence of the series. (ii) Find Laurent series about the indicated singularity for the function $\dfrac{e^z}{(z-1)^2}$; $z = 1$. **[8+7 Marks]**

9. (i) State and prove Cauchy's integral formula. Thus evaluate $\oint_C \dfrac{\cos z}{z - \pi}\,dz$, where C is the circle $|z - 1| = 3$. (ii) State the Residue Theorem and apply it to evaluate $\oint_C \dfrac{e^z}{(z-1)(z+3)^2}\,dz$, where C is given by $|z| = \dfrac{3}{2}$. **[8+7 Marks]**

2022

10. Compute the integral $\oint_C \frac{1+2z+z^2}{(z-1)^2(z+2)}\,dz$, where C is $|z|=3$. **[8 Marks]**

11. Find a bilinear transformation $w=f(z)$ which maps the upper half plane $\mathrm{Im}(z)\geq 0$ onto the unit disk $|w|\leq 1$. **[15 Marks]**

12. (i) Find an upper bound for the absolute value of the integral $I=\int_C e^z\,dz$, where C is the line segment joining the points $(0,0)$ and $(1,3)$. (ii) Find the length of the curve C defined by $z(t)=(1-2it)^3$, $-1\leq t\leq 1$. **[8+7 Marks]**

2021

13. If $f(z)=u+iv$ is any analytic function of the complex variable z and $u-v=e^x(\cos y-\sin y)$, find $f(z)$ in terms of z. **[8 Marks]**

14. Find the Taylor's series expansion of a function of the complex variable $f(z)=\frac{1}{(z-1)(z-3)}$ about the point $z=4$. Find its radius of convergence. **[15 Marks]**

15. Show that $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\,d\theta=\frac{\pi}{12}$. **[15 Marks]**