RAMANASRI
IAS/IFoS Institute

Dynamics, Statics & Hydro Statics

IFoS (IFS) Previous Year Questions (PYQ) from 2025 to 2009

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IAS, UPSC, IFS, IFoS, CIVIL SERVICE MAINS EXAMS MATHS OPTIONAL STUDY MATERIALS

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2025

1. The displacement of a particle in a straight line is given by the equation $x = a\cos nt + b\sin nt$. Show that the particle describes a simple harmonic motion whose amplitude is $\sqrt{a^2+b^2}$ and period is $\dfrac{2\pi}{n}$. **[8 Marks]**

2. A light ladder is supported on a rough floor and leans against a smooth wall. How far up the ladder can a man climb without slipping taking place? **[8 Marks]**

3. A projectile is launched with initial speed u making an angle $(90^\circ - \theta)$ with the vertical. An air resistance force $-\beta\mathbf{v}$, $\beta > 0$, acts upon the projectile, where **v** is the instantaneous velocity. Find the velocity **v**, and show that the position vector **r** at any time t is **[15 Marks]**

$$\mathbf{r} = \frac{mu}{\beta}(\cos\theta\,\hat{\mathbf{j}} + \sin\theta\,\hat{\mathbf{k}})(1 - e^{-\beta t/m}) - \frac{mg}{\beta}\left(t + \frac{m}{\beta}e^{-\beta t/m} - \frac{m}{\beta}\right)\hat{\mathbf{k}},$$

where m is the mass of the projectile.

4. A framework $ABCD$ consists of four equal, light rods smoothly jointed together to form a square. It is suspended from a peg at A, and a weight w is attached to C, the framework being kept in shape by a light rod connecting B and D. Find the thrust in this rod. **[10 Marks]**

5. A solid hemisphere is placed with its base inclined to the surface of a liquid, in which it is completely immersed, at a given angle α. Show that if the resultant thrust on the curved portion of the surface is equal to twice the weight of the liquid displaced, then $\tan\alpha = 2$. **[15 Marks]**

2024

6. A particle moves from rest at a distance a from a centre of force where repulsion at distance x is μx^{-2}. Show that its velocity at distance x is $\sqrt{\dfrac{2\mu(x-a)}{ax}}$ and that the time it has taken is **[8 Marks]**

$$\sqrt{\frac{a}{2\mu}}\left[\sqrt{x^2 - ax} + a\log_e\left(\sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a} - 1}\right)\right].$$

7. Two uniform steel rods of equal size l hang from their junction and rest on a symmetrically placed smooth vertical circular base of radius a. If each of the rods subtends an angle ϕ with the vertical line passing through the centre of the circular base, show that applying the principle of virtual work, the relation obtained is $l = 2a\cot\phi\,\text{cosec}^2\,\phi$. **[8 Marks]**

8. A particle moves in a path so that its acceleration is always directed to a fixed point and is equal to $\mu/(\text{distance})^2$. Show that its path is a conic section and distinguish between the three cases that arise. Further show that the square of the periodic time varies as the cube of the major axis. **[15 Marks]**

9. If near the surface of a celestial body having atmosphere, the gravity is almost constant and the absolute temperature in its atmosphere is given by $T = T_0\sqrt{1 - \dfrac{z^2}{n^2H^2}}$, H being the height of the homogeneous atmosphere and n a constant quantity, show that the pressure in the atmosphere will be given by $p = p_0 \exp\left(\sin^{-1}\dfrac{z}{nH}\right)$, where p_0 is the pressure at $z = 0$. **[10 Marks]**

10. Establish a stability criterion if a rigid body is lying on another rigid body at a point of contact, and also both have rough surface preventing sliding and a small area around the point of contact of both of them is circular. A solid frustum of a paraboloid of revolution of height h and latus rectum $2a$ rests with its vertex on that of a paraboloid of revolution of latus rectum $2b$. Find the stability condition. **[15 Marks]**

2023

11. Four bars are joined together to form a rhombus. The bars are uniform and each bar is of weight W. A rhombus is suspended vertically from one of the joints and a spherical ball of weight S is balanced inside the rhombus so as to keep the system intact. If 2θ is the angle at a fixed joint in the state of equilibrium, then find the ratio of weight of the rhombus to that of the spherical ball in terms of the radius of the sphere, the length of a bar and the angle θ. **[8 Marks]**

12. In a central orbit, the central force is given as $\mu u^3(3 + 2a^2u^2)$. If a particle is projected at a distance a with velocity $\sqrt{\dfrac{5\mu}{a^2}}$ in a direction making an angle $\tan^{-1}\left(\dfrac{1}{2}\right)$ with the radius, then show that equation of its path can be written as $r = a\tan\theta$. **[8 Marks]**

13. A particle slides down the smooth curve $y = a\sinh(x/a)$, the axis of x being horizontal and the axis of y downwards, starting from rest at the point where the tangent is inclined at α to the horizon. Show that the particle will leave the curve when it has fallen through a vertical distance $a\sec\alpha$. **[15 Marks]**

14. A uniform ladder of 10 m length and of 10 kg weight rests with its foot on the rough ground and its upper end against a smooth wall, the inclination to the vertical being α. A force P is applied horizontally to the ladder at a point distant 3 m from the foot, so as to make the foot approach the wall. Prove that the force P must exceed $\dfrac{100}{7}\left(\mu + \dfrac{1}{2}\tan\alpha\right)$, where μ is the coefficient of friction at the foot. **[10 Marks]**

15. An elliptic lamina is completely immersed in water with its plane vertical. Its minor axis is horizontal and is at a depth h. Determine the centre of pressure. **[15 Marks]**

2022

16. Three forces P, Q and R act along the sides BC, CA and AB of triangle ABC in order to keep the system in equilibrium. If the resultant force touches the inscribed circle, then prove that **[8 Marks]**

$$\frac{1+\cos\alpha}{P}+\frac{1+\cos\beta}{Q}+\frac{1+\cos\gamma}{R}=0,$$

where α, β, γ are the interior angles subtended at A, B, C respectively.

17. A person is drawing water from a well with a light bucket which leaks uniformly. The bucket weighs 50 kg when it is full. When it arrives at the top, half of the water remains inside. If the depth of the water level in the well from the top is 30 m, then find the work done in raising the bucket to the top from the water level. **[8 Marks]**

18. A particle of mass m, which is attached to one end of a light string whose other end is fixed at a point O, describes a circular motion in a horizontal plane about the vertical axis through O. Prove that the particle moves in a conical pendulum only if $g < l\omega^2$, where l is the length of the string and ω being angular velocity. Further, a particle of mass m is attached to the middle of a light string of length $2l$, one end of which is fastened to a fixed point and the other end to a smooth ring of mass M which slides on a smooth vertical rod. If the particle describes a horizontal circle with uniform angular velocity ω about the rod, then prove that the inclination of both portions of the string to the vertical is **[15 Marks]**

$$\cos^{-1}\left(\frac{m+2M}{ml\omega^2}\right).$$

19. PR and QR are two equal heavy strings tied together at R and carrying a weight W at R. P and Q are two points in the same horizontal line and $2a$ is the distance between them. l is the length of each string and h is the depth of R below PQ. Prove that (i) $l^2-h^2=2c^2\{\cosh(a/c)-1\}$, (ii) tension at P or $Q=\dfrac{lW+(l^2+h^2)w}{2h}$, where c is the parameter of the catenary and w is the line density of the string. **[15 Marks]**

20. A bucket is in the form of a frustum of a cone and is filled with water of density ρ. If the bottom and top ends of the bucket have radii a and b respectively and h is the height of the bucket, then find the resultant vertical thrust on the curved surface of the bucket. Is that thrust equal to $\dfrac{1}{3}\pi\rho gh(b-a)(b+2a)$? **[10 Marks]**

2021

21. A particle is projected in a direction making an angle α with the horizon. It passes through the two points (x_1, y_1) and (x_2, y_2). Prove that **[8 Marks]**

$$\tan\alpha = \frac{y_1 R}{Rx_1 - x_1^2} = \frac{x_2^2 y_1 - x_1^2 y_2}{x_1 x_2 (x_2 - x_1)},$$

where R denotes the horizontal range.

22. Four light rods are joined smoothly to form a quadrilateral $ABCD$. Let P and Q be the mid-points of an opposite pair of rods and these points are connected by a string in a state of tension T. Let R and S be the mid-points of the other opposite pair of rods and these points are connected by a light rod in a state of thrust X. Show that $T \cdot RS = X \cdot PQ$. **[8 Marks]**

23. A particle is moving in a medium with central acceleration P. The medium is a resisting medium in which resistance $= kv^2$, v being the velocity. Let s be the arc-length; (r, θ) be plane polar coordinates; $u = 1/r$ and M_0 be the initial moment of momentum about the centre of force. Show that the equation of the path of the particle is **[15 Marks]**

$$Pe^{2ks} = M_0^2 u^2 \left(u + \frac{d^2u}{d\theta^2}\right).$$

24. Given a solid in the shape of a double cone bounded by two equal circular ends. The solid floats in a liquid, whose density is twice that of the cone, with its axis horizontal. Prove that the equilibrium is stable or unstable according as the semi-vertical angle is less than or greater than $60°$. **[15 Marks]**

25. If the mass density at any point of a cord varies as the radius of curvature of the curve in which it hangs freely under gravity, then prove that this curve is the catenary of uniform strength. **[10 Marks]**