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Linear Algebra

Previous year Questions from 2025 to 1992

2025

1. Can the set $\{(0,0,0,3),(1,1,0,0),(0,1,-1,0)\}$ be extended to form a basis of the vector space R^4? Justify your answer.

[10 Marks]

2. Find the range, rank, kernel and nullity of the linear transformation $T: R^4 \to R^3$ given by $T(x,y,z,w) = (x-w, y+z, z-w)$.

[10 Marks]

3. Let $T: R^3 \to R^2$ be a linear transformation such that $T(1,1,-1) = (1,0)$, $T(4,1,1) = (0,1)$ and $T(1,-1,2) = (1,1)$. Find T.

[15 Marks]

4. Reduce the following matrix to echelon form :

$$A = \begin{bmatrix} 2 & -2 & 2 & 1 \\ -3 & 6 & 0 & -1 \\ 1 & -7 & 10 & 2 \end{bmatrix}$$

[15 Marks]

5. Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & -2 & 3 \end{bmatrix}$$

[12 Marks]

6. Let P_n denote the vector space of all polynomials of degree $\leq n$ over R. Verify that $\dim(P_4/P_2) = \dim P_4 - \dim P_2$.

[8 Marks]

2024

7. Let H be a subspace of R^4 spanned by the vectors $v_1 = (1,-2,5,-3)$, $v_2 = (2,3,1,-4)$, $v_3 = (3,8,-3,-5)$. Then find a basis and dimension of H, and extend the basis of H to a basis of R^4.

[10 Marks]

8. Let $T: R^3 \to R^3$ be a linear operator and $B = \{v_1, v_2, v_3\}$ be a basis of R^3 over R. Suppose that $Tv_1 = (1,1,0)$, $Tv_2 = (1,0,-1)$, $Tv_3 = (2,1,-1)$. Find a basis for the range space and null space of T.

[10 Marks]

9. Consider the linear operator T on R^3 over R defined by $T(x,y,z) = (2x, 4x-y, 2x+3y-z)$. Is T invertible? If yes, justify your answer and find T^{-1}.

[15 Marks]

10. Let $V = M_{2\times 2}(R)$ denote a vector space over the field of real numbers. Find the matrix of the linear mapping $\phi: V \to V$ given by $\phi(v) = Av$ with respect to the standard basis of $M_{2\times 2}(R)$, where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

Hence find rank ϕ. Is ϕ invertible? Justify your answer.

[15 Marks]

11. Let

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

be a 3×3 matrix. Find the eigenvalues and the corresponding eigenvectors of A. Hence find the eigenvalues and the corresponding eigenvectors of A^{-15}, where $A^{-15} = (A^{-1})^{15}$.

[20 Marks]

2023

12. Let $V_1 = (2,-1,3,2)$, $V_2 = (-1,1,1,-3)$ and $V_3 = (1,1,9,-5)$ be three vectors of the space R^4. Does $(3,-1,0,-1) \in \text{span}\{V_1,V_2,V_3\}$? Justify your answer.
[10 Marks]
13. Find the rank and nullity of the linear transformation $T : R^3 \to R^3$ given by $T(x,y,z) = (x+z, x+y+2z, 2x+y+3z)$.
[10 Marks]
14. If the matrix of a linear transformation $T : R^3 \to R^3$ relative to the basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ is
$$\begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$
then find the matrix of T relative to the basis $\{(1,1,1),(0,1,1),(0,0,1)\}$.
[15 Marks]
15. Let
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$
(i) Verify the Cayley-Hamilton theorem for the matrix A. (ii) Show that $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$, where I is the identity matrix of order 3. Hence find A^{40}.
[10 + 10 Marks]
16. Find the rank of the matrix
$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$
by reducing it to row-reduced echelon form.
[15 Marks]

2022

17. Prove that any set of n linearly independent vectors in a vector space V of dimension n constitutes a basis for V.
[10 Marks]
18. Let $T : R^2 \to R^3$ be a linear transformation such that $T(1,0) = (1,2,3)$ and $T(1,1) = (-3,2,8)$. Find $T(2,4)$.
[10 Marks]
19. Find all solutions to the following system of equations by row-reduced method :
$$\begin{aligned} x_1 + 2x_2 - x_3 &= 2, \\ 2x_1 + 3x_2 + 5x_3 &= 5, \\ -x_1 - 3x_2 + 8x_3 &= -1. \end{aligned}$$
[15 Marks]
20. Let the set
$$P = \{(x,y,z)^T : x - y - z = 0 \text{ and } 2x - y + z = 0\}$$
be the collection of vectors of a vector space R^3. Then (i) prove that P is a subspace of R^3, (ii) find a basis and dimension of P.
[10 + 10 Marks]
21. Find a linear map $T : R^2 \to R^2$ which rotates each vector of R^2 by an angle θ. Also, prove that for $\theta = \dfrac{\pi}{2}$, T has no eigenvalue in R.
[15 Marks]

2021

22. If

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

then show that $A^2 = A^{-1}$ without finding A^{-1}.

[10 Marks]

23. Find the matrix associated with the linear operator on $V_3(R)$ defined by $T(a,b,c) = (a+b, a-b, 2c)$ with respect to the ordered basis $B = \{(0,1,1),(1,0,1),(1,1,0)\}$.

[10 Marks]

24. Show that $S = \{(x, 2y, 3z) : x, y, z$ are real numbers$\}$ is a subspace of R^3. Find two bases of S. Also find the dimension of S.

[15 Marks]

25. Prove that the eigenvectors corresponding to two distinct eigenvalues of a real symmetric matrix are orthogonal.

[8 Marks]

26. For two square matrices A and B of order 2, show that trace(AB) = trace(BA). Hence show that $AB - BA \neq I_2$, where I_2 is the identity matrix of order 2.

[7 Marks]

27. Reduce the following matrix to a row-reduced echelon form and hence also find its rank :

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$$

[10 Marks]

28. Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

over the complex-number field.

[10 Marks]

2020

29. Consider the set V of all $n \times n$ real magic squares. Show that V is a vector space over R. Give examples of two distinct 2×2 magic squares.

[10 Marks]

30. Let $T : M_2(R)$ be the vector space of all 2×2 real matrices. Let $B = \begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix}$.

Suppose $T : M_2(R) \to M_2(R)$ is a linear transformation defined by $T(A) = BA$.

Find the rank and nullity of T. Find a matrix A which maps to the null matrix.

[10 Marks]

31. Define an $n \times n$ matrix as $A = I - 2u.u^T$, where u is a unit column vector

(i) Examine if A is symmetric.

(ii) Examine if A is orthogonal.

(iii) Show that trace $(A) = n - 2$.

(iv) Find $A_{3\times3}$ when $u = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$ **[20 Marks]**

32. Let F be a subfield of complex numbers and T a function from $F^3 \to F^3$ defined by $T(x_1,x_2,x_3) = (x_1 + x_2 + 3x_3, 2x_1 - x_2, -3x_1 + x_2 - x_3)$. What are the conditions on a,b,c such that (a,b,c) be in the null space of T? find the nullity of T. **[15 Marks]**

33. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$

(i) Find AB

(ii) Find det (A) and det (B)

(iii) Solve the following system of linear equations:

$x + 2z = 3 \quad 2x - y + 3z = 3 \quad 4x + y + 8z = 14$ **[15 Marks]**

2019

34. Let $T: R^2 \to R^2$ be a linear map such that $T(2,1) = (5,7)$ and $T(1,2) = (3,3)$ If A is the matrix corresponding to T with respect to the standard bases $e_1, e_2,$ then find rank **[10 Marks]**

35. If $A\begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$ and $B\begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$ then show that $AB = 6I_3$. Use this result to solve the following system of equations. $2x + y + z = 5,\ x - y = 0,\ 2x + y - z = 1$ **[10 Marks]**

36. Let A and B be two orthogonal matrices of same order and det $A+$ det $B = 0$ Show that $A + B$ is a singular matrix. **[15 Marks]**

37. Let $A = \begin{pmatrix} 5 & 7 & 2 & 1 \\ 1 & 1 & -8 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 4 & -3 & 1 \end{pmatrix}$

(i) Find the rank of matrix A

(ii) Find the dimension of the subspace $V = \left\{ (x_1, x_2, x_3, x_4) \in R^4 \;\middle|\; A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0 \right\}$ **[15+5=20 Marks]**

38. State the Cayley-Hamilton theorem. Use this theorem of find A^{100} where $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **[15 Marks]**

2018

39. Let A be a 3×2 matrix and B a 2×3 matrix. Show that $C = AB$ is a singular matrix. **[10 Marks]**

40.

41. Express basis vectors $e_1 = (1,0)$ and $e_2 = (0,1)$ as linear combinations of $\alpha_1 = (2,-1)$ and $\alpha_2 = (1,3)$. **[10 Marks]**

42. Show that if A and B are similar $n\times n$ matrices, then they have the same Eigen values. **[12 Marks]**

43. For the system of linear equations $x+3y-2z=-1,\ 5y+3z=-8,\ x-2y-5z=7$ determine which of the following statements are true and which are false:

(i) The system has no solution.

(ii) The system has a unique solution.

(iii) The system has infinitely many solutions. **[13 Marks]**

2017

44. Let $A=\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find a non-singular matrix P such that $P^{-1}AP$ is diagonal matrix. **[10 Marks]**

45. Show that similar matrices have the same characteristic polynomial. **[10 Marks]**

46. Suppose U and W are district four dimensional subspaces of a vector space $V,$ where dim $V=6.$ Find the possible dimensions of subspace $U\bigcap W$ **[10 Marks]**

47. Consider the matrix mapping $A:R^4 \to R^3,$ where $A=\begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{pmatrix}$. Find a basis and dimension of the image of A and those of the kernel A. **[15 Marks]**

48. Prove that distance non-zero eigenvectors of a matrix are linearly independent. **[10 Marks]**

49. Consider the following system of equation in x, y, z $x+2y+2z=1,\ x+ay+3z=3,\ x+11y+az=b$

(i) For which values of a does the system have a unique?

(ii) For which of values (a,b) does the system have more than one solution? **[15 Marks]**

2016

50. (i) Using elementary row operations, find the inverse of $A=\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ **[6 Marks]**

(ii) If $A=\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ then find $A^{14}+3A-2I.$ **[4 Marks]**

51. (i) Using elementary row operation find the condition that the linear equations have a solution

$$x - 2y + z = a$$
$$2x + 7y - 3z = b$$ **[7 Marks]**
$$3x + 5y - 2z = c$$

(ii) If $W_1 = \{(x,y,z) | x + y - z = 0\}$, $W_2 = \{(x,y,z) | 3x + y - 2z = 0\}$, $W_3 = \{(x,y,z) | x - 7y + 3z = 0\}$ then find $\dim(W_1 \cap W_2 \cap W_3)$ and $\dim(W_1 + W_2)$ **[3 Marks]**

52. (i) If $M_2(R)$ is space of real matrices of order 2×2 and $P_2(x)$ is the space of real polynomials of degree at most 2, then find the matrix representation of $T : M_2(R) \to P_2(x)$ such that $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + b + c + (a - d)x + (b + c)x^2$, with respect to the standard bases of $M_2(R)$ and $P_2(x)$ further find null space of T **[10 Marks]**

(ii) If $T : P_2(x) \to P_3(x)$ is such that $T(f(x)) = f(x) + 5\int_0^x f(t)dt,$ then choosing $\{1, 1+x, 1-x^2\}$ and $\{1, x, x^2, x^3\}$ as bases of $P_2(x)$ and $P_3(x)$ respectively find the matrix of T. **[6 marks]**

53. (i) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the Eigen values and Eigenvectors of A. **[6 Marks]**

(ii) Prove that Eigen values of a Hermitian matrix are all real. **[8 Marks]**

54. If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ is the matrix representation of a linear transformation $T : P_2(x) \to P_2(x)$ with respect to the bases $\{1 - x, x(1 - x), x(1 + x)\}$ and $\{1, 1 + x, 1 + x^2\}$ then find T. **[18 Marks]**

2015

55. The vectors $V_1 = (1,1,2,4), V_2 = (2,-1,-5,2), V_3 = (1,-1,-4,0)$ and $V_4 = (2,1,1,6)$ are linearly independent. Is it true? Justify your answer. **[10 Marks]**

56. Reduce the following matrix to row echelon form and hence find its rank:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$$

[10 Marks]

57. If matrix $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$ then find A^{30} **[12 Marks]**

58. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$ **[12 Marks]**

59. Let $V=R^3$ and $T\in A(V)$, for all $a_i\in A(V)$, be defined by $T(a_1,a_2,a_3)=(2a_1+5a_2+a_3,-3a_1+a_2-a_3,a_1+2a_2+3a_3)$. What is the matrix T relative to the basis $V_1=(1,0,1), V_2=(-1,2,1), V_3=(3,-1,1)$? **[12 Marks]**

60. Find the dimension of the subspace of R^4, spanned by the set $\{(1,0,0,0),(0,1,0,0),(1,2,0,1),(0,0,0,1)\}$. Hence find its basis. **[12 Marks]**

2014

61. Find one vector in R^3 which generates the intersection of V and W, where V is the xy-plane and W, is the space generated by the vectors $(1,2,3)$ and $(1,-1,1)$ **[10 Marks]**

62. Using elementary row or column operations, find the rank of the matrix

$$\begin{bmatrix}0&1&-3&-1\\0&0&1&1\\3&1&0&2\\1&1&-2&0\end{bmatrix}$$

[10 Marks]

63. Let V and W be the following subspaces of R^4 : $V=\{(a,b,c,d):b-2c+d=0\}$ and $W=\{(a,b,c,d):a=d,b=2c\}$. Find a basis and the dimension of (i) V (ii) W (iii) $V\cap W$ **[15 Marks]**

64. Investigate the values of λ and μ so that the equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have (i) no solution (ii) unique solution, (iii) an infinite number of solutions. **[10 Marks]**

65. Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1&4\\2&3\end{bmatrix}$ and hence find its inverse. Also, find the matrix represented by $A^5-4A^4-7A^3+11A^2-A-10I$ **[10 Marks]**

66. Let $A=\begin{bmatrix}-2&2&-3\\2&1&-6\\-1&-2&0\end{bmatrix}$. Find the Eigen values of A and the corresponding Eigen vectors. **[8 Marks]**

67. Prove that Eigen values of a unitary matrix have absolute value 1. **[7 Marks]**

2013

68. Find the inverse of the matrix: $A=\begin{bmatrix}1 & 3 & 1\\ 2 & -1 & 7\\ 3 & 2 & -1\end{bmatrix}$ by using elementary row operations. Hence solve the system of linear equations $x+3y+z=10$ $2x-y+7z=12$ $3x+2y-z=4$ **[10 Marks]**

69. Let A be a square matrix and A^* be its ad joint, show that the Eigen values of matrices AA^* and A^*A are real. Further show that $\text{trace}(AA^*)=\text{trace}(A^*A)$ **[10 Marks]**

70. Let P_n denote the vector space of all real polynomials of degree at most n and $T:P_2\to P_3$ be linear transformation given by $T(f(x))=\int_0^x p(t)\,dt,\ p(x)\in P_2$. Find the matrix of T with respect to the bases $\{1,x,x^2\}$ and $\{1,x,1+x^2,1+x^3\}$ of P_2 and P_3 respectively. Also find the null space of T **[10 Marks]**

71. Let V be an n- dimensional vector space and $T:V\to V$ be an invertible linear operator. If $\beta=\{X_1,X_2,...X_n\}$ is a basis of V, show that $\beta'=\{TX_1,TX_2,...TX_n\}$ is also a basis of V **[8 Marks]**

72. Let $A=\begin{bmatrix}1 & 1 & 1\\ 1 & \omega^2 & \omega\\ 1 & \omega & \omega^2\end{bmatrix}$ where $\omega(\neq 1)$ is a cube root of unity. If $\lambda_1,\lambda_2,\lambda_3$ denote the Eigen values of A^2, show that $|\lambda|_1+|\lambda_2|+|\lambda_3|\leq 9$ **[8 Marks]**

73. Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3 & 4 & 5\\ 2 & 3 & 5 & 8 & 12\\ 3 & 5 & 8 & 12 & 17\\ 3 & 5 & 8 & 17 & 23\\ 8 & 12 & 17 & 23 & 30\end{bmatrix}$ **[8 Marks]**

74. Let A be a Hermitian matrix having all distinct Eigen values $\lambda_1,\lambda_2,....\lambda_n$. If $X_1,X_2,...X_n$ are corresponding Eigen vectors then show that the $n\times n$ matrix C whose k^{th} column consists of the vector X_n is non-singular. **[8 Marks]**

75. Show that the vectors $X_1=(1,1+i,i)$, $X_2=(i,-i,1-i)$ and $X_3=(0,1-2i,2-i)$ in C^3 are linearly independent over the field of real numbers but are linearly dependent over the field of complex numbers. **[8 Marks]**

2012

76. Prove or disprove the following statement: If $B=\{b_1,b_2,b_3,b_4,b_5\}$ is a basis for $\mathbb{R}^5$ and V is a two-dimensional subspace of $\mathbb{R}^5$, then V has a basis made of two members of B. **[12 Marks]**

77. Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be the linear transformation defined by $T(\alpha,\beta,\gamma)=(\alpha+2\beta-3\gamma,2\alpha+5\beta-4\gamma,\alpha+4\beta+\gamma)$. Find a basis and the dimension of the image of T and the kernel of T **[12 Marks]**

78. Let V be the vector space of all 2×2 matrices over the field of real numbers. Let W be the set consisting of all matrices with zero determinant. Is W a subspace of V? Justify your answer? **[8 Marks]**

79. Find the dimension and a basis for the space W of all solutions of the following homogeneous system using matrix notation:

$$x_1+2x_2+3x_3-2x_4+4x_5=0$$
$$2x_1+4x_2+8x_3+x_4+9x_5=0$$
$$3x_1+6x_2+13x_3+4x_4+14x_5=0$$

[12 Marks]

80. (i) Consider the linear mapping $f:\mathbb{R}^2 \to \mathbb{R}^2$ by $f(x,y)=(3x+4y,2x-5y)$. Find the matrix A relative to the basis $(1,0),(0,1)$ and the matrix B relative to the basis $(1,2),(2,3)$ **[12 Marks]**

(ii) If λ is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\lambda}$ is a characteristic root of $\text{Adj } A$ [8 Marks]

81. Let $H=\begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$ be a Hermitian matrix. Find a non-singular matrix P such that $D=P^T H\bar{P}$ is diagonal. **[20 Marks]**

2011

82. Let A be a non-singular $n\times n$, square matrix. Show that $A.\ (adjA)=|A|.I_n$ Hence show that $|adj(adjA)|=|A|^{(n-1)^2}$ **[10 Marks]**

83. Let $A=\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}, B=\begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$ Solve the system of equations given by $AX=B$ Using the above, also solve the system of equations $A^T X=B$ where A^T denotes the transpose of matrix A.
[10 Marks]

84. Let $\lambda_1,\lambda_2,.....\lambda_n$ be the Eigen values of a $n\times n$ square matrix A with corresponding Eigen vectors $X_1,X_2,.....X_n$. If B is a matrix similar to show that the Eigen values of B is same as that of A. Also find the relation between the Eigen vectors of B and Eigen vectors of A.
[10 Marks]

85. Show that the subspaces of $\mathbb{R}^3$ spanned by two sets of vectors $\{(1,1,-1),(1,0,1)\}$ and $\{(1,2,-3),(5,2,1)\}$ are identical. Also find the dimension of this subspace. **(10Marks)**

86. Find the nullity and a basis of the null space of the linear transformation $A:\mathbb{R}^{(4)} \to \mathbb{R}^{(4)}$ given by the matrix $A=\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. **[10 Marks]**

87. Show that the vectors $(1,1,1),(2,1,2)$ and $(1,2,3)$ are linearly independent in $\mathbb{R}^{(3)}$. Let $\mathbb{R}^{(3)} \to \mathbb{R}^{(3)}$ be a linear transformation defined by $T(x,y,z)=(x+2y+3z,x+2y+5z,2x+4y+6z)$ Show that the images of above vectors under are linearly dependent. Given the reason for the same.

(ii)Let $A=\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and C be a non-singular matrix of order 3×3. Find the Eigen values of the matrix B^3 where $B=C^{-1}AC$. **[10 Marks]**

2010

88. If $\lambda_1, \lambda_2, ...\lambda_3$ are the Eigen values of the matrix $A = \begin{bmatrix} 26 & -2 & 2 \\ 2 & 21 & 4 \\ 44 & 2 & 28 \end{bmatrix}$ show that $\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} \leqq \sqrt{1949}$

[12 Marks]

89. What is the null space of the differentiation transformation $\frac{d}{dx} : P_n \to P_n$ where P_n is the space of all polynomials of degree $\leq n$ over the real numbers? What is the null space of the second derivative as a transformation of? What is the null space of the kith derivative P_n?

[12 Marks]

90. Let. $M = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ Find the unique linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ so that M is the matrix of T with respect to the basis $\beta = \{v_1 = (1,0,0) v_2 = (1,1,0) v_3 = (1,1,1)\}$ of $\mathbb{R}^3$ and $\beta' = \{w_1 = (1,0), w_2 = (1,1)\}$ of $\mathbb{R}^2$ .Also find $T(x, y, z)$.

[20 Marks]

91. Let A and B be $n \times n$ matrices over reals. Show that BA is invertible if $I - AB$ is invertible. Deduce that AB and AB have the same Eigen values. **[20 Marks]**

92. (i) In the space R^n determine whether or not the $\{e_1 - e_2, e_2 - e_3, ..., e_{n-1} - e_n, e_n - e_1\}$ set is linearly independent.

(ii) Let T be a linear transformation from a vector V space over reals into V such that $T - T^2 = I$ Show that is invertible. **[20 Marks]**

2009

93. Find a Hermitian and skew-Hermitian matrix each whose sum is the matrix. $\begin{bmatrix} 2i & 3 & -1 \\ 1 & 2+3i & 2 \\ -i+1 & 4 & 5i \end{bmatrix}$

[12 Marks]

94. Prove that the set V of the vectors (x_1, x_2, x_3, x_4) in which $\mathbb{R}^4$ satisfy the equation $x_1 +, x_2 + x_3 + x_4 = 0$ and $2x_1 + 3x_2 - x_3 + x_4 = 0$, is a subspace of $\mathbb{R}^4$ .What is dimension of this subspace? Find one of its bases. **[12 Marks]**

95. Let $\beta = \{(1,1,0)(1,01)(0,1,1)\}$ and $\beta' = \{(2,1),(1,2,1)(-1,1,1)\}$ be the two ordered bases of R^3. Then find a matrix representing the linear transformation $T : R^3 \to R^3$ which transforms β into β'. Use this matrix representation to find $T(x)$, where $x = (2,3,1)$. **[20 Marks]**

96. Find a 2×2 real matrix A which is both orthogonal and skew-symmetric. Can there exist a 3×3 real matrix which is both orthogonal and skew-symmetric? Justify your answer. **(20Marks**

97. Let $L : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by $L = (x_1, x_2, x_3, x_4))$ $= (x_3 + x_4 - x_1 - x_2, x_3 - x_2, x_4 - x_1)$. Then find the rank and nullity of L. Also, determine null space and range space of L. **[20 Marks]**

98. Prove that the set V of all 3×3 real symmetric matrices form a linear subspace of the space of all 3×3 real matrices. What is the dimension of this subspace? Find at least of the bases for V.

[20 Marks]

2008

99. Show that the matrix A is invertible if and only if the $adj(A)$ is invertible. Hence find $|adj(A)|$

[12 Marks]

100. Let S be a non-empty set and let V denote the set of all functions from S into R. Show that V is vector space with respect to the vector addition $(f+g)(x)=f(x)+g(x)$ and scalar multiplication $(c.f)(x)=cf(x)$ **[12 Marks]**

101. Show that $B=\{(1,0,0),(1,1,0),(1,1,1)\}$ is a basis of. R^3 Let $T:R^3\to R^3$ be a linear transformation such that $T(1,0,0)=(1,0,0), T(1,.1,0)=(1,1,1)$ and $T(1,1,1)=(1,1,0)$ .Find $T(x,y,z)$

[15 Marks]

102. Let A be a non-singular matrix. Show that if $I+A+A^2+.....+A^n=0$ then $A^{-1}=A^n$.

[15 Marks]

103. Find the dimension of the subspace of R^4 spanned by the set $\{(1,0,0,0)(0,1,0,0)(1,2,0,1),(0,0,0,1)\}$. Hence find a basis for the subspace.

[15 Marks]

2007

104. Let S be the vector space of all polynomials, $p(x)$ with real coefficients, of degree less than or equal to two considered over the real field $|R$ such that $p(0)$ and $p(1)=0$ .Determine a basis for S and hence its dimension.

[12 Marks]

105. Let T be the linear transformation from $|R^3$ to $|R^4$ define by $T(x_1,x_2,x_3)=(2x_1+x_2+x_{3,}x_1,x_{2,}x_1+x_3,3x_1+x_2=2x_3)$ for each $(x_1,x_2,x_3)\in|R^3$ Determine a basis for the Null space of T. What is the dimension of the Range space of T?

[12 Marks]

106. Let W be the set of all 3×3 symmetric matrices over $|R$ does it from a subspace of the vector space of the 3×3 matrices over $|R$? In case it does, construct a basis for this space and determined its dimension **[15 Marks]**

107. Consider the vector space $X:=\{p(x)\}$ is a polynomial of degree less than or equal to 3 with real coefficients. Over the real field $|R$ define the map $D:X\to X$ by $(Dp)(x):=P_1+2P_2x+3P_3x^2$ where $p(x):=P_0+P_1x+P_2x^2+p_3x^3$ is D a linear transformation on X? If it is then construct the matrix representation for D with respect to the order basis $\{1,x,x^2,x^3\}$ for X. **[15 Marks]**

108. Reduce the quadratic form $q(x,y,z):=x^2+2y^2-4xz-4yz+7z^2$ to canonical form. Ss positive definite? **[15 Marks]**

2006

109. Let V be the vector space of all 2×2 matrices over the field F. Prove that V has dimension 4 by exhibiting a basis for V. **[12 Marks]**

110. State Cayley-Hamilton theorem and using it, find the inverse of $\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$. **[12 Marks]**

111. If $T:R^2\to R^2$ is defined by $T(x,y)=(2x-3y,x+y)$ compute the matrix of T relative to the basis $\beta\{(1,2),(2,3)\}$ **[15 Marks]**

112. Using elementary row operations, find the rank of the matrix $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$.

[15 Marks]

113. Investigate for what values of and the equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

Have-

(i) no solution;

(ii) a unique solution;

(iii) infinitely many solutions **[15 Marks]**

114. Find the quadratic form $q(x,y)$ corresponding to the symmetric matrix $A=\begin{pmatrix} 5 & -3 \\ -3 & 8 \end{pmatrix}$ Is this quadratic from positive definite? Justify your answer. **[15 Marks]**

2005

115. Find the values of k for which the vectors $(1,1,1,1),(1,3,-2,k),(2,2k-2,-k-2,3k-1)$ and $(3,k+2,-3,2k+1)$ are linearly independent in R^4. **[12 Marks]**

116. Let V be the vector space of polynomials in x of degree $\le n$ over R. Prove that the set $\{1,x,x^2,...,x^n\}$ is a basis for the set of all polynomials in x. **[12 Marks]**

117. Let T be a linear transformation on R^3 whose matrix relative to the standard basis of R^3 is $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ Find the matrix of T relative to the basis $\beta=\{(1,1,1),(1,1,0),(01,1)\}$. **[15 Marks]**

118. Find the inverse of the matrix given below using elementary row operations only:

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

[15 Marks]

119. If S is a skew-Hermitian matrix, then show that is a unitary matrix. Also show that $A=(I+S)(I-S)^{-1}$ every unitary matrix can be expressed in the above form provided -1 is not an Eigen value of A. **[15 Marks]**

120. Reduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ to the sum of squares. Also find the corresponding linear transformation, index and signature.

[15 Marks]

121. Let S be space generated by the vectors $\{(0,2,6),(3,1,6),(4,-2,-2)\}$ what is the dimension of the space S? Find a basis for S. **[12 Marks]**

2004

122. Show that $f: R^3 \to IR$ is la linear transformation, where $f(x,y,z) = 3x + y - z$ what is the dimension of the Kernel? Find a basis for the Kernel.

123. Show that the linear transformation form IR^3 to IR^4 which is represented by the matrix $\begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & -2 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ is one-to-one. Find a basis for its image. **[12 Marks]**

124. Verify whether the following system of equation is consistent

$$x + 3z = 5$$
$$-2x + 5y - z = 0$$
$$-x + 4y + z = 4$$

[15 Marks]

125. Find the characteristic polynomial of the matrix $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$ Hence find A^{-1} and A^6 **(15Marks)**

126. Define a positive definite quadratic form. Reduce the quadratic form to canonical form. Is this quadratic form positive definite? **[15 Marks]**

2003

127. Let S be any non-empty subset of a vector pace V over the field F. Show that the set $\{a_1\alpha_1 + a_2\alpha_2 + ... + a_n\alpha_{n:} : a_1, a_2, ..., a_n \in F, \alpha_1, \alpha_2, ..., \alpha_n \in S, n \in N\}$ is the subspace generated by S. **[12 Marks]**

128. If $= \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ then find the matrix represented by $2A^{10-}10A^9 + 14A^8 - 6A^7 - 3A^6 + 15A^5 - 21A^4 + 9A^3 + A - 1.$ **[12 Marks]**

129. Prove that the Eigen vectors corresponding to distinct Eigen values of a square matrix are linearly independent. **[15 Marks]**

130. If H is a Hermitian matrix, then show that $A = (H + iI)^{-1}(H - iI)$ is a unitary matrix. Also, so that every unitary matrix can be expressed in this form, provided 1 is not an Eigen value of A. **[15 Marks]**

131. If $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ then find a diagonal matrix D and a matrix B such that $A = BDB'$ where B' denotes the transpose of B. **[15 Marks]**

132. Reduce the quadratic form given below to canonical form and find its rank and signature $x^2 + 4y^2 + 9z^2 + u^2 - 12yz + 6zx - 4xy - 2xu - 6zu.$ **[15 Marks]**

2002

133. Show that the mapping $T: R^3 \to R^3$ where $T(a,b,c) = (a-b, b-c, a+c)$ is linear and non-singular **[12 Marks]**

134. A square matrix A is non-singular if and only if the constant term in its characteristic polynomial is different from zero. **[12 Marks]**

135. Let $R^5 \to R^5$ be a linear mapping given by $T(a,b,c,d,e) = (b-d, +e, b, 2d+e, b+e)$ Obtain based for its null space and range space. **[15 Marks]**

136. Let A be a real 3×3 symmetric matrix with Eigen values0, 0 and 5 If the corresponding Eigen-vectors are $(2,0,1),(2,1,1)$ and $(1,0,-2)$ then find the matrix A.

[15 Marks]

137. Solve the following system of linear equations
$$x_1-2x_2-3x_3+4x_4=-1$$
$$-x_1+3x_2+5x_3-5x_4-2x_5=0$$
$$2x_1+x_2-2x_3+3x_4-4x_5=17,$$
[15 Marks]

138. Use Cayley-Hamilton theorem to find the inverse of the following matrix: $\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$ **[15 Marks]**

2001

139. Show that the vectors $(1,0-1),(0,-3,2)$ and $(1,2,1)$ form a basis for the vector space $R^3(R)$

[12 Marks]

140. If λ is a characteristic root of a non-singular matrix A then prove that $\frac{|A|}{\lambda}$ is a characteristic root of $Adj.A$ **[12 Marks]**

141. If $A=\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix}$ show that for every integer $n\geq3, A^n=A^{n-2}+A^2-I$ Hence determine A^{50}.

[15 Marks]

142. When is a square matrix A said to be congruent to a square matrix B? Prove that every matrix congruent to skew-symmetric matrix is skew symmetric. **[15 Marks]**

143. Determine an orthogonal matrix P such that is a diagonal matrix, where $=\begin{pmatrix}7&4&4\\4&-8&-1\\-4&-1&-8\end{pmatrix}$ **[15 Marks]**

144. Show that the real quadratic form $\phi=n\left(x_1^2+x_2^2+\ldots+x_n^2\right)-\left(x_1x_2+\ldots+x_n\right)^2$ in n variables is positive semi-definite. **[15 Marks]**

2000

145. Let V be a vector space over R and $T=\{(x,y)|x,y,\in v\}$ Let. Define addition in component wise and scalar multiplication by complex number $\alpha+i\beta$ by $(\alpha+i\beta)(x,y)=(\alpha x+\beta y,\beta y+\alpha y)\forall\alpha\beta\in R$ Show that T is a vector space over C. **[12 Marks]**

146. Show that if λ is a characteristic root of a non-singular matrix A then λ^{-1} is a characteristic root of A^{-1} **[15 Marks]**

147. Prove that a real symmetric matrix A is positive definite if and only $A=BB^t$ if for some non-singular matrix. B Show also that $A=\begin{pmatrix}1&2&3\\2&5&7\\3&7&11\end{pmatrix}$ is positive definite and find the matrix B such that $A=BB^t$

Here stands for the transpose of. **[15 Marks]**

148. Prove that a system $AX=B$ if non-homogeneous equations in unknowns have a unique solution provided the coefficient matrix is non-singular. **[15 Marks]**

149. Prove that two similar matrices have the same characteristic roots. Is its converse true? Justify your claim. **[15 Marks]**

150. Reduce the equation $x^2 + y^2 + z^2 - 2xy - 2yz + 2zx + x - y - 2z + 6 = 0$ into canonical form and determine the nature of the quadratic. **[15 Marks]**

1999

151. Let V be the vector space of functions from R to R (the real numbers). Show that f, g, h in V are linearly independent where $f(t) = e^{2t}, g(t) = t^2$ and $h(t) = t$. **[20 Marks]**

152. If the matrix of a linear transformation T on $V_2(R)$ with respect to the basis, then what is the matrix of with respect to the ordered basis $B = \{(1,0),(0,1)\}$ is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then what is the matrix of T with respect to the ordered basis. **[20 Marks]**

153. Diagonalize the matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$ **[20 Marks]**

154. Test for congruency of the matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and. $B = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ Prove that $A^{2n} = B^{2m} I$ when and are positive integers. **[20 Marks]**

155. If A is askew symmetric matrix of order n Prove that $(I - A)(I + A)^{-1}$ is orthogonal. **[20 Marks]**

156. Test for the positive definiteness of the quadratic form $2x^2 + y^2 + 2z^2 - 2zx$. **[20 Marks]**

1998

157. Given two linearly independent vectors $(1,0,1,0)$ and $(0,-1,1,)$ of R^4 find a basis of which included these two vectors **[20 Marks]**

158. If is a finite dimensional vector space over R and if and are two linear transformations from V to R such that $Vf(v) = 0$ in plies $g(v) = 0$ then prove that $g = \lambda f$ form some λ in R. **[20 Marks]**

159. Let $T : R^3 \to R^3$ be defined by $T(x_1, x_2, x_3) = (x_2, x_3 - cx_1 bx_2 - ax_3)$ where a, b, c are fixed real numbers. Show that T is a linear transformation of R^3 and that $A^3 + aA^2 + ba + = cI = 0$ where A is the matrix of T with respect to standard basis of R^3 **[20 Marks]**

160. If A and B are two matrices of order 2×2 such that A is skew Hermitian and $AB = B$ then show that $B = 0$ **[20 Marks]**

161. If T is a complex matrix of order 2×2 such that $trT = trT^2 = 0$ then show that $T^2 = 0$ **[20 Marks]**

162. Prove that a necessary and sufficient condition for a $n \times n$ real matrix to be similar to a diagonal matrix A is that the set of characteristic vectors A of includes a set of linearly independent vectors. **[20 Marks]**

163. Let be a matrix. Then show that the sum of the rank and nullity of A is n. **[20 Marks]**

164. Find all real 2×2 matrices A whose characteristic roots are real and which satisfy $A\,A' = 1$ **(20Marks)**

165. Reduce to diagonal matrix by rational congruent transformation the symmetric matrix
$$A=\begin{pmatrix}1&2&-1\\2&0&3\\-1&3&1\end{pmatrix}.$$ **[20 Marks]**

1997

166. Let V be the vector space of polynomials over R. Find a basis and dimension of the subspace W of V spanned by the polynomials
$v_1=t^3-2t^2+4t+1, v_2=2t^3-3t^2+9t-1, v_3=t^3+6t-5, v_4=2t^3-5t^2+7t+5$ **[20 Marks]**

167. Verify that the transformation defined by $T(x_1,x_2)=(x_1+x_2,x_1-x_2,x_2)$ is a linear transformation from R^2 into R^3. Find its range, null space and nullity. **[20 Marks]**

168. Let V be the vector space of 2×2 matrices over R. Determine whether the matrices $A,B,C\in V$ are dependent where $A=\begin{bmatrix}1&2\\3&1\end{bmatrix}, B=\begin{bmatrix}3&-1\\2&2\end{bmatrix}, C=\begin{bmatrix}1&-5\\-4&0\end{bmatrix}$ **[20 Marks]**

169. Let a square matrix A of order n be such that each of its diagonal elements is μ and each of its off-diagonal elements is 1. If $B=\lambda A$ is orthogonal, determined the values of λ and μ **[20 Marks]**

170. Show that $A=\begin{bmatrix}2&-1&0\\-1&2&0\\2&2&3\end{bmatrix}$ is diagonalizable over R and find a matrix P such that $P^{-1}AP$ is diagonal. Hence determine A^{25} **[20 Marks]**

171. Let $A=[a_{ij}]$ be a square matrix of order n such that $[a_{ij}]\le M\ \forall i,j=1,2,...n$. Let λ be an Eigen-value of A. Show that $|\lambda|\le nM$ **[20 Marks]**

172. Define a positive definite matrix. Show that a positive definite matrix is always non-singular. Prove that its converse does not hold. **[20 Marks]**

173. Find the characteristics roots and their corresponding vectors for the matrix
$$\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$$ **[20 Marks]**

174. Find an invertible matrix P which reduces $Q(x,y,z)=2xy+2yz+2zx$ to its canonical form. **[20 Marks]**

1996

175. R^4, let W_1 be the space generated by $(1,1,0,-1),(2,6,0)$ and $(-2,-3,-3,1)$ and let W_2 be the space generate by $(-1,-2,-2,2),(4,6,4,-6)$ and $(1,3,4,-3)$. Find a basis for the space W_1+W_2 **[20 Marks]**

176. Let V be a finite dimensional vector space and $v\in V, v\neq 0$. Show that there exist a linear functional f on V such that V **[20 Marks]**

177. Let $V = R^3$ and v_1, v_2, v_3 be a basis of R^3. Let $T : V \to V$ be a linear transformation such that. By writing the matrix of T with respect to another basis, show that the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is similar to $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **[20 Marks]**

178. Let $V = R^3$ and $T : V \to V$ be linear map defined by $T(x, y, z) = (x + z, -2x + y, -x + 2y + z)$. What is the matrix of T with respect to the basis $(1,0,1), (-1,1,1)$ and $(0,1,1)$? Using this matrix, write down the matrix of T with respect to the basis $(0,1,2), (-1,1,1)$ and $(0,1,1)$ **[20 Marks]**

179. Let V and W be finite dimensional vector spaces such that $\dim V \geq \dim W$. Show that there is always a linear map from V onto W **[20 Marks]**

180. Solve

$$x + y - 2z = 1$$
$$2x - 7z = 3$$
$$x + y - z = 5$$

by using Cramer's rule **[20 Marks]**

181. Find the inverse of the matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

by computing its characteristic polynomial. **[20 Marks]**

182. Let A and B be $n \times n$ matrices such that $AB = BA$. Show that A and B have a common characteristic vector. **[20 Marks]**

183. Reduce to canonical form the orthogonal matrix $\begin{bmatrix} 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}$ **[20 Marks]**

1995

184. Let T be the linear operator in R^3 defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$. What is the matrix of T in the standard ordered basis of R^3? What is a basis of range space of T and a basis of null space of T? **[20 Marks]**

185. Let A be a square matrix of order n. Prove that $AX = b$ has solution if and only if $b \in R^n$ is orthogonal to all solutions Y of the system $A^T Y = 0$ **[20 Marks]**

186. Define a similar matrix. Prove that the characteristic equation of two similar matrices is the same. Let 1, 2, and 3 be the Eigen-values of a matrix. Write down such a matrix. Is such a matrix unique? **[20 Marks]**

187. Show that $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is diagonalizable and hence determine A^5. **[20 Marks]**

188. Let A and B be matrices of order n. Prove that if $(I - AB)$ is invertible, then $(I - BA)$ is also invertible and $(I - BA)^{-1} = I + B(I - AB)^{-1}A$. Show that AB and BA have precisely the same characteristic values. **[20 Marks]**

189. If a and b complex numbers such that and H is a Hermitian matrix, show that the Eigen values of lie on a straight line in the complex plane. **[20 Marks]**

190. Let A be a symmetric matrix. Show that A is positive definite if and only if its Eigen values are all positive. **[20 Marks]**

191. Let A and B be square matrices of order n. Show that AB-BA can never be equal to unit matrix. **[20 Marks]**

192. Let A and for every. Show that A is a non-singular matrix. Hence or otherwise prove that the Eigen-values of A lie in the discs in the complex plane. **[20 Marks]**

1994

193. Show that $f_1(t)=1,\ f_2(t)=t-2,\ f_3(t)=(t-2)^2$ form a basis of P_3, the space of polynomials with degree ≤ 2. Express $3t^2-5t+4$ as a linear combination of f_1, f_2, f_3. **[20 Marks]**

194. If $T: V_4(R) \rightarrow V_3(R)$ is a linear transformation defined by $T(a,b,c,d)=(a-b+c+d, a+2c-d, a+b+3c-3d)$. For $a,b,c,d \in R$, then verify that $\text{Rank } T + \text{Nullity } T = \dim V_4(R)$ **[20 Marks]**

195. If T is an operator on R_3 whose basis is $B=\{(1,0,0),(0,1,0),(-1,1,0)\}$ such that $[T:B]=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$ find the matrix T with respect to a basis $B_1=\{(0,1,-1),(1,-1,1),(-1,1,0)\}$ **[20 Marks]**

196. If $A=[a_{ij}]$ is an $n\times n$ matrix such that $a_{ii}=n, a_{ij}=r$ if $i\neq j$, show that $[A-(n-r)I][A-(n-r+nr)I]=0$. Hence find the inverse of the $n\times n$ matrix $B=[b_{ij}]$. where $b_{ii}=1, b_{ij}=\rho$ when $i\neq j$ and $\rho\neq 1, \rho\neq \frac{1}{1-n}$ **[20 Marks]**

197. Prove that the Eigen vectors corresponding to distinct Eigen values of a square matrix are linearly independent. **[20 Marks]**

198. Determine the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ **[20 Marks]**

199. Show that a matrix congruent to a skew-symmetric matrix is skew-symmetric. Use the result to prove that the determinant of skew-symmetric matrix of even order is the square of a rational function of its elements. **[20 Marks]**

200. Find the rank of the matrix $\begin{bmatrix} 0 & c & -b & a' \\ -c & 0 & a & b' \\ b & -a & 0 & c' \\ -a' & -b' & -c' & 0 \end{bmatrix}$ where $aa'+bb'+cc'=0$ a,b,c are all positive integers **[20 Marks]**

201. Reduce the following symmetric matrix to a diagonal form and interpret the result in terms of quadratic forms: $A=\begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$ **[20 Marks]**

1993

202. Show that the set $S=\{(1,0,0),(1,1,0),(1,1,1),(0,1,0)\}$ spans the vector space $R^3(R)$ but it is not a basis set. **[20 Marks]**

203. Define rank and nullity of a linear transformation T. If V be a finite dimensional vector space and T a linear operator on V such that rank $T^2 =$ rank T, then prove that the null space of $T =$ the null space of T^2 and the intersection of the range space and null space to T is the zero subspace of V. **[20 Marks]**

204. If the matrix of a linear operator T on R^2 relative to the standard basis $\{(1,0),(0,1)\}$ is $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, what is the matrix of T relative to the basis $B=\{(1,1),(1,-1)\}$? **[20 Marks]**

205. If A be an orthogonal matrix with the property that -1 is not an Eigen value, then show that a is expressible as $(I-S)(S+S)^{-S1}$ for some suitable skew-symmetric matrix S. **[20 Marks]**

206. Determine the following form as definite, semi-definite or indefinite:
$2{x_1}^2+2{x_2}^2+3{x_3}^2-4x_2x_3-4x_3x_1+2x_1x_2$ **[20 Marks]**

207. Prove that the inverse of $\begin{pmatrix} A & O \\ B & C \end{pmatrix}$ is $\begin{pmatrix} A^{-1} & O \\ C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$ where A, C are non-singular matrices and hence find the inverse of $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ **[20 Marks]**

208. Show that any two Eigen vectors corresponding to two distinct Eigen values of Hermitian matrix and Unitary matrix are orthogonal **[20 Marks]**

209. A matrix B of order $n\times n$ is of the form λA where λ is a scalar and A has unit elements everywhere except in the diagonal which has elements μ. Find λ and μ so that B may be orthogonal. **[20 Marks]**

210. Find the rank of the matrix $\begin{pmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{pmatrix}$ by reducing it to canonical form. **[20 Marks]**

1992

211. Let V and U be vector spaces over the field K and let V be of finite dimension. Let $T:V\to U$ be a linear Map. $\dim V=\dim R(T)+\dim N(T)$ **[20 Marks]**

212. Let $S=\{(x,y,z)/x+y+z=0\}$, x,y,z being real. Prove that S is a subspace of R^3. Find a basis of S **[20 Marks]**

213. Verify which of the following are linear transformations?
(i) $T:R\to R^2$ defined by $T(x)=(2x,-x)$
(ii) $T:R^2\to R^3$ defined by $T(x,y)=(xy,y,x)$
(iii) $T:R^2\to R^3$ defined by $T(x,y)=(x+y,y,x)$
(iv) $T:R\to R^2$ defined by $T(x)=(1,-1)$ **[20 Marks]**

214. Let $T:M_{2,1}\to M_{2,3}$ be a linear transformation defined by (with usual notations)
$T\begin{pmatrix} 1 \\ 0 \end{pmatrix}=\begin{pmatrix} 2 & 1 & 3 \\ 4 & 1 & 5 \end{pmatrix}$, $T\begin{pmatrix} 1 \\ 1 \end{pmatrix}=\begin{pmatrix} 6 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ Find $T\begin{pmatrix} x \\ y \end{pmatrix}$ **[20 Marks]**

215. For what values of η do the following equations

$x+y+z=1$

$x+2y+4z=\eta$ Have solutions? Solve them completely in each case. **[20 Marks]**

$x+4y+10z=\eta^2$

216. Prove that a necessary and sufficient condition of a real quadratic form $X'AX$ to be positive definite is that the leading principal minors of A are all positive. **[20 Marks]**

217. State Cayley-Hamilton theorem and use it to calculate the inverse of the matrix $\rightarrow\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ **[20 Marks]**

218. Transform the following to the diagonal forms and give the transformation employed:
$x^2+2y,\ 8x^2-4xy+5y^2$ **[20 Marks]**

219. Prove that the characteristic roots of a Hermitian matrix are all real and a characteristic root of a skew-Hermitian is either zero or a pure imaginary number. **[20 Marks]**