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UPSC MATHS OPTIONAL COACHING

Mechanics &

Fluid Dynamics

Previous year Questions

from 2025 to 1992

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2025

1. A bead of mass m slides on a frictionless wire in the shape of a cycloid given by $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $0 \le \theta \le 2\pi$. Find the Lagrangian function. Hence show that the equation of motion can be written as **[10 Marks]**

$$\frac{d^2u}{dt^2} + \frac{g}{4a}u = 0,$$

where $u = \cos\left(\frac{\theta}{2}\right)$.

2. A source and a sink of equal strength are placed at the points $\left(\pm\frac{a}{2}, 0\right)$ within a fixed circular boundary $x^2 + y^2 = a^2$. Show that the streamlines are given by **[10 Marks]**

$$\left(r^2 - \frac{a^2}{4}\right)(r^2 - 4a^2) - 4a^2y^2 = ky(r^2 - a^2),$$

where k is a constant and $r^2 = x^2 + y^2$.

3. Calculate the moment of inertia of a uniform solid cylinder of mass M, radius R and length L with respect to a set of axes passing through the centre of the cylinder, where z-axis is the axis of the cylinder and ρ is the constant density at any point of the cylinder. Also find L/R for which the moment of inertia about x- or y-axis will be minimum for a given mass of the cylinder. **[15 Marks]**

4. Show that for an incompressible steady flow with constant viscosity, the velocity components **[20 Marks]**

$$u(y) = \left(\frac{U}{h}\right)y - \frac{hy}{2\mu}\frac{dp}{dx}\left(1 - \frac{y}{h}\right), \qquad v = 0 = w,$$

with $p = p(x)$, satisfy the equation of motion in the absence of body force. Given that U, h and $\frac{dp}{dx}$ are constants.

5. (i) A particle of mass m moves in a force field of potential **[10 Marks]**

$$V(r) = \frac{k\cos\theta}{r^2},$$

where k is constant. Find the Hamiltonian and Hamilton's equations in spherical polar coordinates (r, θ, ϕ).
(ii) Consider the Lagrangian $L = m\dot{x}\dot{y} - m\omega_0^2 xy$, where m and ω_0 are constants. Find the Hamiltonian and Hamilton's equations of motion. Identify the system.

2024

6. A rough uniform board of mass m and length $2a$ rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Find the distance covered by the board during this time. **[10 Marks]**

7. The velocity potential ϕ of a flow is given by **[10 Marks]**

$$\phi = \frac{1}{2}(x^2 + y^2 - 2z^2).$$

Determine the streamlines.

8. Find the moment of inertia of a quadrant of an elliptic disk **[15 Marks]**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

of mass M, about the line passing through its centre and perpendicular to its plane. Given that the density at any point is proportional to xy.

9. Let the velocity field **[20 Marks]**

$$u(x, y) = \frac{B(x^2 - y^2)}{(x^2 + y^2)^2}, \qquad v(x, y) = \frac{2Bxy}{(x^2 + y^2)^2}, \qquad w(x, y) = 0$$

satisfy the equations of motion for inviscid incompressible flow, where B is a constant. Determine the pressure associated with this velocity field.

10. Suppose an infinite liquid contains two parallel, equal and opposite rectilinear vortices at a distance $2a$. Show that the streamlines relative to the vortex are given by **[20 Marks]**

$$\log \frac{x^2 + (y - a)^2}{x^2 + (y + a)^2} + \frac{y}{a} = C,$$

where C is a constant, the origin is the middle point of the join, and the line joining the vortices is the axis of y.

2023

11. A planet of mass m is revolving around the sun of mass M. The kinetic energy T and the potential energy V of the planet are given by **[10 Marks]**

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2), \qquad V = GMm\left(\frac{1}{2a} - \frac{1}{r}\right),$$

where (r, θ) are the polar coordinates of the planet at time t, G is the gravitational constant and $2a$ is the major axis of the ellipse. Find the Hamiltonian and the Hamilton equations of the planet's motion.

12. In a fluid motion, there is a source of strength $2m$ placed at $z = 2$ and two sinks of strength m are placed at $z = 2 + i$ and $z = 2 - i$. Find the streamlines. **[10 Marks]**

13. A mechanical system with 2 degrees of freedom has the Lagrangian **[20 Marks]**

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}m(\omega_1^2 x^2 + \omega_2^2 y^2) + kxy,$$

where m, ω_1, ω_2, k are constants. Find the parameter θ so that under the transformation

$$x = q_1 \cos\theta - q_2 \sin\theta, \qquad y = q_1 \sin\theta + q_2 \cos\theta,$$

the Lagrangian in terms of q_1, q_2 will not contain the product term $q_1 q_2$. Find the Lagrange's equations with respect to q_1 and q_2 independent of parameter θ.

14. A perfectly rough ball is at rest within a hollow cylindrical roller. The roller is drawn along a level path with uniform velocity V. Let a and b be the radii of the ball and the roller respectively. If **[15 Marks]**

$$V^2 > \frac{27}{7}g(b-a),$$

then show that the ball will roll completely round the inside of the roller.

15. Determine under what conditions the velocity field $u = c(x^2 - y^2)$, $v = -2cxy$, $w = 0$ is a solution of the Navier-Stokes momentum equations. Assuming the conditions are met, determine the resulting pressure distribution, when z is up and the external body forces are $B_x = 0 = B_y$, $B_z = -g$. **[20 Marks]**

2022

16. A particle at a distance r from the centre of force moves under the influence of the central force **[10 Marks]**

$$F = -\frac{k}{r^2},$$

where k is a constant. Obtain the Lagrangian and derive the equations of motion.

17. The velocity components of an incompressible fluid in spherical polar coordinates (r, θ, ψ) are **[10 Marks]**

$$(2Mr^{-3}\cos\theta,\ Mr^{-2}\sin\theta,\ 0),$$

where M is a constant. Show that the velocity is of the potential kind. Find the velocity potential and the equations of the streamlines.

18. Find the moment of inertia of a right circular solid cone about one of its slant sides (generator), in terms of its mass M, height h and the radius of base a. **[15 Marks]**

19. Two point vortices each of strength k are situated at $(\pm a, 0)$ and a point vortex of strength $-k/2$ is situated at the origin. Show that the fluid motion is stationary and also find the equations of streamlines. If the streamlines, which pass through the stagnation points, meet the x-axis at $(\pm b, 0)$, then show that **[20 Marks]**

$$3\sqrt{3}\,(b^2 - a^2)^2 = 16a^3 b.$$

20. Verify that **[20 Marks]**

$$w = ik \log\left(\frac{z - ia}{z + ia}\right)$$

is the complex potential of a steady flow of fluid about a circular cylinder, where the plane $y = 0$ is a rigid boundary. Find also the force exerted by the fluid on unit length of the cylinder.

2021

21. A particle is constrained to move along a circle lying in the vertical xy-plane. With the help of D'Alembert's principle, show that its equation of motion is **[10 Marks]**

$$x\ddot{y} - y\ddot{x} - gx = 0,$$

where g is the acceleration due to gravity.

22. What arrangements of sources and sinks can have the velocity potential **[10 Marks]**

$$w = \log_e\left(z - \frac{a^2}{z}\right)?$$

Draw the corresponding sketch of the streamlines and prove that two of them subdivide into the circle $r = a$ and the axis of y.

23. Obtain the Lagrangian equation for the motion of a system of two particles of unequal masses connected by an inextensible string passing over a small smooth pulley. **[15 Marks]**

24. Show that **[20 Marks]**

$$\vec{q} = \frac{\lambda(-y\hat{i} + x\hat{j})}{x^2 + y^2}, \qquad \lambda = \text{constant},$$

is a possible incompressible fluid motion. Determine the streamlines. Is the kind of the motion potential? If yes, then find the velocity potential.

25. Show that for the complex potential $\tan^{-1} z$, the streamlines and equipotential curves are circles. Find the velocity at any point and check the singularities at $z = \pm i$. **[20 Marks]**

2020

26. Prove that the moment of inertia of a triangular lamina ABC about any axis through in A its plane is $\frac{M}{6}(\beta^2+\beta\gamma+\gamma^2)$ where M is the mass of the lamina and β,γ are respectively the length of perpendiculars from B and C on the axis. **[10 Marks]**

27. By writing down the Hamiltonian, find the equations of motion of a particle of mass m constrained to move on the surface of a cylinder defined by $x^2+y^2=R^2$ is a constant. The particle is subject to a force directed towards the origin and proportional to the distance r of the particle from the origin given by $\vec{F}=-k\vec{r}, k$ is a constant **[20 Marks]**

28. A velocity potential in a two-dimensional fluid flow is given by $\phi(x,y)=xy+x^2-y^2$ find the stream function for this flow. **[15 Marks]**

29. One end of a tightly stretched flexible thin string of length l is fixed at the origin and the other at $x=l$. It is plucked at $x=\frac{1}{3}$ so that it assumes initially the shape of a triangle of height h in the $x-y$ plane. Find the displacement y at any distance x and at any time t after the string is released from rest. Take, $\frac{\text{horizontal tension}}{\text{mass per unit length}}=c^2$. **[20 Marks]**

30. Two sources of strength $\frac{m}{2}$ are placed at the points $(\pm a,0)$. Show that at any point. Show that at any point on the circle $x^2+y^2=a^2$ the velocity is parallel to the y axis and is inversely proportional to y. **[15 Marks]**

2019

31. A uniform rod OA of length $2a$ free to turn about it end O revolves with angular velocity about the vertical OZ through O and is inclined at a constant angle α to OZ; find the value of α **[10 Marks]**

32. A circular cylinder of radius and radius a of gyration k rolls without slipping inside a fixed hollow cylinder of radius b. Show that the plane thorough axes move in a circular pendulum of length $(b-a)\left(1+\frac{k^2}{a^2}\right)$. **[20 Marks]**

33. Using Hamilton's equation find the acceleration for a sphere rolling down a rough inclined plane, if x be the distance of the point of contact of the sphere from a fixed point on the plane. **[15 Marks]**

34. A sphere of radius R whose centre is at rest, vibrates radially in an infinite incompressible fluid of density ρ, which is at rest at infinity, if the pressure at infinity is Π, so that the pressure at the surface of the sphere at time t is $\Pi+\frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2}+\left(\frac{dR}{dt}\right)^2\right\}$. **[15 Marks]**

35. Two sources, each of strength are places at the points $(-a,0),(a,0)$ and a sink of strength at origin. Show that the stream lines are the curves $(x^2+y^2)^2=a^2(x^2-y^2+\lambda xy)$ where λ is a variable parameter. Show also that the fluid speed at any point is $(2a^2)/(r_1r_2r_3)$ where r_1r_2 and r_1 re the distances of the points from the source and sings respectively. **[20 Marks]**

2018

36. For an incompressible fluid flow, two components of velocity (u,v,w) are given by $u = x^2 + 2y^2 + 3z^2$ $v = x^2 y - y^2 z + zx$. Determine the third component w so that they satisfy the equation of continuity. Also, find the z-component of acceleration. **[10 Marks]**

37. Suppose the Lagrangian of a mechanical system is given by $L = \frac{1}{2}m(ax^2 + 2bxy + cy^2) - \frac{1}{2}k(ax^2 + 2bxy + cy^2)$, Where $a, b, c,\ m(>0), k(>0)$ are constants and $b^2 \neq ac.$ write down the Lagrangian equations of motion and identify the system. **[20 Marks]**

38. The Hamiltonian of a mechanical system is given by, $H = p_1 q_1 - aq_1^2 + bq_2^2 - p_2 q_2$ Where a, b are the constants. Solve the Hamiltonian equations and show that $\frac{p_2 - bq_2}{q_1} =$ constant. **[20 Marks]**

39. For a two-dimensional potential flow, the velocity potential is given by $\phi = x^2 y - xy^2 + \frac{1}{3}(x^3 - y^3)$. Determine the velocity components along the directions x and y. Also, determine the stream function ψ and check whether ϕ represents a possible case of flow or not. **[15 Marks]**

40. A thin annulus occupies the region. $0 \leq a \leq r \leq b,\ 0 \leq \theta \leq 2\pi$ The faces are insulated, Along the inner edge the temperature is maintained 0^0, while along the outer edge the temperature is held at $T = K\cos\frac{\theta}{2}$ where K is a constant. Determine the temperature distribution in the annulus. **[20 Marks]**

2017

41. Show that the moment if inertia of an elliptic area of mass M and semi-axis a and b about a semi-diameter of length r is $\frac{1}{4}M\frac{a^2b^2}{r^2}$. Further, prove that the moment of inertia about a tangent is $\frac{5M}{4}p^2$ where p is the perpendicular distance from the centre of the ellipse to the tangent. **[10 Marks]**

42. Two uniform rods AB, AC each of mass m and length $2a,$ are smoothly hinged together at A and move on a horizontal plane. At time t, the mass centre of the road is at the point (ξ, η) referred to fixed perpendicular axes Ox, Oy in the plane, and the rods make angles $\theta \pm \phi$ with $Ox.$ Prove that the kinetic energy of the system is $m\left[\xi^2 + \eta^2 + \left(\frac{1}{3} + \sin^2\phi\right)a^2\theta^2 + \left(\frac{1}{3} + \cos^2\phi\right)a^2\phi^2\right]$. Also derive Lagrange's equation of motion for the system if an external force with components $[X, Y]$ along the axes acts at A. **[20 Marks]**

43. A stream is rushing form a boiler through a conical pipe, the diameters of the ends of which are D and d. If V and v be the corresponding velocities of the stream and if the motion is assumed to steady and diverging from the vertex of the cone, then prove that $\frac{v}{V} = \frac{D^2}{d^2}e^{(u^2 - v^2)/2K}$, where K is the pressure divided by the density and is constant. **[15 Marks]**

44. If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$, $r^2 = x^2 + y^2 + z^2$, then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$. Further, determine the streamlines. **[15 Marks]**

2016

45. Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r}\right]$ possess vorticity, where $\vec{q}(u,v,w)$ is the velocity in the Cartesian frame $\vec{r}(x,y,z)$ and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle $x^2 + y^2 = 9, z = 0$? **[10 marks]**

46. Consider single free particle of mass m, moving in space under no forces. If the particle starts form the origin $t = 0$ at and reaches the position (x, y, z) at time τ, find the Hamilton's characteristic function S as a function of x, y, z, τ. **[10 marks]**

47. A simple source of strength m is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity $U\vec{i}$ Show that he velocity potential ϕ at any point P of the stream is $\frac{m}{r} - Ur\cos\theta$, where $OP = r$ and θ is the angle which $\overrightarrow{OP}$ makes with the direction $\vec{i}$. Find the deferential equation of the streamlines and show that the lie on the surfaces $Ur^2\sin^2\theta - 2m\cos\theta =$ constant. **[15 marks]**

48. The space between two concentric spherical shells of radii a, b $(a < b)$ is filled with a liquid of density ρ. If the shells are set in motion, the inner one with velocity U in the x- direction and the outer one with velocity V in the y- direction, then show has the initial motion of the liquid is given by velocity potential

$$\phi = \frac{\left\{a^3U\left(1 + \frac{1}{2}b^3r^{-3}\right)x - b^3V\left(1 + \frac{1}{2}a^3r^{-3}\right)y\right\}}{(b^3 - a^3)}$$

, where $r^2 = x^2 + y^2 + z^2$, the coordinate being rectangular. Evaluate the velocity at any point of the liquid. **[20 marks]**

49. A hoop with radius r is rolling, without slipping, down an inclined plane of length l and with angle of inclination ϕ. Assign appropriate generalized coordinate to the system. Determine the constraints, if any. Write down the Lagrangian equation for the system. Hence or otherwise determine the velocity of the hoop at the bottom of the inclined plane. **[15 marks]**

2015

50. Consider a uniform flow U_0 in the positive x- direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity positional. Find also the stagnation points. **[10 Marks]**

51. Calculate the moment of inertia of a solid uniform hemisphere $x^2 + y^2 + z^2 = a^2,\ z \geq 0$ with mass m about the OZ- axis. **[10 Marks]**

52. Solve the plane pendulum problem using the Hamiltonian approach and show that H is a constant of motion. **[15 Marks]**

53. A Hamiltonian of a system with one degree of freedom has form

$$H = \frac{p^2}{2\alpha} - bape^{-\alpha t} + \frac{b\alpha}{2}q^2e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{k}{2}q^2$$

where α, b, k are constant, q is the generalized coordinate and p is the corresponding generalized momentum.

(i) Find a Lagrangian corresponding too this Hamiltonian.

(ii) Find an equivalent Lagrangian that is not explicitly dependent on time. **[10+10=20 Marks]**

54. In an axis symmetric motion, show that stream function exists due to equation of continuity. Express the velocity components of the stream function. Find the equation satisfied by the stream function if the flow is irrotational. **[20 Marks]**

2014

55. Find the equation of motion of a compound pendulum using Hamilton's equations. **[10 Marks]**

56. Given the velocity potential $\phi = \frac{1}{2}\log\left[\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}\right]$, determine the streamlines. **[20 Marks]**

57. Find Navier-Stokes equation for steady laminar flow of a viscous incompressible fluid between two infinite parallel plates. **[20 Marks]**

2013

58. Prove that the necessary and sufficient conditions that the vortex lines may be at right angles to the stream lines are $u, v, w = \mu\left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$ where μ and ϕ are function of x, y, z, t. **[10 Marks]**

59. For solid sphere A, B, C and D, each of mass m and radius a, are placed with their centers on the four corners of a square of side b. Calculate the moment of inertia of the system about a diagonal of the square. **[10 Marks]**

60. Two equal rod AB and BC, each of length l, smoothly jointed at B, are suspended from A and oscillate in a vertical plane through A. Show that that the periods of normal oscillation are $\frac{2\pi}{n}$ where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$ **[15 Marks]**

61. If fluid fills the region of space on the positive side of the x- axis, which is a right boundary and if there be a sources m at the point $(0, a)$ and an equal sink at $(0, b)$ and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is $\frac{\pi\rho m^2 (a-b)^2}{\{2ab(a+b)\}}$ where ρ is the density of the fluid. **[15 Marks]**

62. If n rectilinear vortices of the same strength K are symmetrically arranged as generators of a circle cylinder of radius a in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{8\pi^2 a^2}{(n-1)\mathrm{K}}$. Find the velocity at any point of the liquid. **[20 Marks]**

2012

63. Obtain the equations governing the motion of a spherical pendulum. **[12 Marks]**

64. A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream. **[12 Marks]**

65. A pendulum consists of a rod of length $2a$ and mass m; to one end of which a spherical bob of radius $\frac{a}{3}$ and mass 15m is attached. Find the moment of inertia of the pendulum:
(i) About an axis through the other end of the rod and at right angle to the rod. **[15 Marks]**
(ii) About a parallel axis through the centre of mass of the pendulum. [Given: the centre of mass of the pendulum is $\frac{a}{12}$ above the centre of the sphere]. **[15 Marks]**

66. Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $\vec{q} \to 0$ as $r \to \infty$, find the surfaces of constant speed. **[30 Marks]**

2011

67. Let a be the radius of the base of a right circular cone of height h and mass M. Find the moment of inertia of that right circular cone about a line through the vertex perpendicular to the axis. **[12 Marks]**

68. The ends of a heavy rod of length $2a$ are rigidly attached to two light rings which can respectively slide on the thin smooth fixed horizontal and vertical wires O_x and O_y. The rod starts at an angle α to the horizon with an angular velocity $\sqrt{\left[3g(1-\sin\alpha)/2a\right]}$ and moved downwards. Show that it will strike the horizontal wire at the end of time $-2\sqrt{a/(3g)}\log\left[\tan\left(\frac{\pi}{8}-\frac{\alpha}{4}\right)\cot\frac{\pi}{8}\right]$. **[30 Marks]**

69. An infinite row of the equidistance rectilinear vortices are at distance a apart. The vortices are of the same numerical strength K but they are alternately of opposite signs. Find the Complex function that determines the velocity potential and the stream function. **[30 Marks]**

2010

70. A uniform lamina is bounded by a parabolic are of latus rectum $4a$ and a double ordinate at a distance b from the vertex. If b $b=\frac{a}{3}\left(7+4\sqrt{7}\right)$, show that two of the principal axes at the end of a latus rectum are the tangent and normal there. **[12 Marks]**

71. In an incompressible fluid the vorticity at every point is constant in magnitude and direction; show that the components of velocity u, v, w are solution of Laplace's equation. **[12 Marks]**

72. A sphere of radius a land mass m rolles down a rough plane inclined at an angle α to the horizontal. If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equation. **[30 Marks]**

73. When a pair of equal and opposite rectilinear vortices are situated in a long circle cylinder at equal distance from its axis, show that the path of each vortex is given by the equation $(r^2\sin^2\theta-b^2)(r^2-a^2)^2=4a^2b^2r^2\sin^2\theta,\ \theta$ being measured from the line through the centre perpendicular to the joint of the vortices. **[30 Marks]**

2009

74. The flat surface of a hemisphere of radius r is cemented to one flat surface of a cylinder of the same radius and of the same material. If the length of the cylinder be l and the total mass be m, show that the moment of inertia of the combination about the axis of the cylinder is given by: $mr^2\dfrac{\left(\frac{l}{2}+\frac{4}{15}r\right)}{\left(l+\frac{2r}{3}\right)}$. **[12 Marks]**

75. Two sources, each o strength m are placed at the point $(-a,0),(a,0)$ and a sink of strength $2m$ is at the origin. Show that the stream lines are the curves: $(x^2+y^2)^2=a^2(x^2-y^2+\lambda xy)$ where λ is a variable parameter. Show also that the fluid speed at any point is $(2ma^2)(r_1r_2r_3)$ where r_1r_2 and r_3 are the distance of the points from the source and the sink. **[12 Marks]**

76. A perfectly rough sphere of mass m and radius b, rests on the lowest point of a fixed spherical cavity a radius a. To the highest point of the movable sphere is attached a particle of mass m' and the system is

disturbed. Show that the oscillations are the same as of a simple pendulum of length $(a-b)\dfrac{4m'+\frac{7}{5}m}{m+m'\left(2-\frac{a}{b}\right)}$.

[30 Marks]

77. An infinite mass of fluid is acted on by a force $\dfrac{\mu}{r^{3/2}}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $r=C$ in it, show that the cavity will be filled up after an interval of time $\left(\dfrac{2}{5\mu}\right)^{\frac{1}{2}}.C^{\frac{5}{4}}$. **[30 Marks]**

2008

78. A circular board is placed on a smooth horizontal plane and a boy runs round the edge of it at a uniform rate. What is the motion of the centre of the board? Explain. What happens if the mass of the board and boy are equal? **[12 Marks]**

79. If the velocity potential of a fluid is $\phi=\dfrac{1}{r^3}z\tan^{-1}\left(\dfrac{y}{z}\right)$, $r^2=x^2+y^2+z^2$ then show that the stream lines lie on the surfaces $x^2+y^2+z^2=c(x^2+y^2)^{2/3}$, c being a constant. **[12 Marks]**

80. A uniform road of mass $3m$ and length $2l$ has its middle point fixed and a mass m is attached to one of its extremity. The rod, when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity $\sqrt{\dfrac{28}{l}}$. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}(\sqrt{2}-1)$. **[30 Marks]**

81. Let the fluid fills the region $x\geq 0$ (right half of $2d$ plane). Let a source α be $(0, y_1)$ and equal sink at $(0, y_2)$, $y_1 > y_2$. Let the pressure be same as pressure infinity i.e., p_0. Show that the resultant pressure on the boundary (y- axis) is $\pi\rho\alpha^2(y_1-y_2)^2/2y_1y_2(y_1+y_2)$, ρ being the density of the fluid. **[30 Marks]**

2007

82. Consider a system with two degree of freedom for which $V=q_1^2+3q_1q_2+4q_2^2$. Find the equilibrium position and determine if the equilibrium is stable. **[12 Marks]**

83. Show that $\left(\dfrac{x^2}{a^2}\right)\cos^2 t+\left(\dfrac{y^2}{b^2}\right)\sec^2 t=1$ is a possible form for the boundary surface of a liquid. **[12 Marks]**

84. A point mass m is placed on a frictionless plane that is tangent to the Earth's surface. Determine Hamilton's equations by taking x or θ as the generalized coordinate. **[30 Marks]**

85. A thin plate of very large area is placed in a gap of height h with oils of viscosities μ' and μ'' on the two sides of the plate. The plate is pulled at a constant velocity V . Calculate the position of the plate so that
(i)The shear force on the sides of the two sides of the plate is equal
(ii) The force required to drag the plate is minimum. [End effects are neglected] **[30 Marks]**

2006

86. Given points $\mathrm{A}(0,0)$ and $\mathrm{B}(x_0, y_0)$ not in the same vertical, it is required to find a curve in the $x-y$ plane joining A to B so that a particle starting from rest will traverse from A to B along this curve without friction in the shortest possible time. If $y = y(x)$ is the required curve find the function $f(x, y, z)$ such that equation of motion can be written as $\frac{dx}{dt} = f\left(x, y(x), y'(x)\right)$. **[12 Marks]**

87. A steady Inviscid incompressible flow has a velocity field $u = fx, v = -fy, w = 0$, where f is a constant. Derive an expression for the pressure field $p(x, y, z)$, if the pressure $p(0,0,0) = p_0$ and $F = -giz$.
[12 Marks]

88. A particle of mass m is constrained to move on the surface of a cylinder. The particle is subject to a force directed towards the origin and proportional to the distance to of particle from the origin. Construct the Hamiltonian and Hamilton's equations of motion. **[30 Marks]**

89. Liquid is contained between two parallel planes; the free surface is a circular cylinder of radius a whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius b is suddenly annihilated; prove that if p be the pressure at the outer surface, the initial pressure at any point on the liquid distant r from the centre is $p\frac{\log r - \log b}{\log a - \log b}$. **[30 Marks]**

2005

90. A rectangular plate swings in a vertical plane about one of its corners. If its period is one second, find the length of its diagonal. **[12 Marks]**

91. Prove that the necessary and sufficient condition for vortex lines and stream to be at right angles to each other is that $u = \mu\frac{\partial\phi}{\partial x}, v = \mu\frac{\partial\phi}{\partial y}, w = \mu\frac{\partial\phi}{\partial z}$ where μ and ϕ are functions of x, y, z and t. **[12 Marks]**

92. A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time. $\sqrt{\frac{2M'a}{\left(M+M'\right)g\sin\alpha}}$ Where a is the length of the plank?
[30 Marks]

93. State the conditions under which Euler's equation of motion can be integrated show that $-\frac{\partial\phi}{\partial t} + \frac{1}{2}q^2 + V\int\frac{dp}{\rho} = F\left(t\right)$ where the symbols have their usual meaning. **[30 Marks]**

2004

94. A particle of mass m moves under the influence of gravity on the inner surface of the paraboloid of revolution $x^2 + y^2 = az$ which is assumed frictionless. Obtain the equation of motion show that it will describe a horizontal circle in the plane $z = h$, provided that it is given an angular velocity whose magnitude is $\omega = \sqrt{\frac{2g}{a}}$. **[12 Marks]**

95. In an incompressible fluid the vorticity at every point is constant in magnitude and direction Do the velocity components? Justify. **[12 Marks]**

96. Derive the Hamilton equations of motion from the principle of least action and obtain the same for a particle of mass m moving in a force field of potential V. Write these equations in spherical coordinates (r, θ, ϕ).
[30 Marks]

97. The space between two infinitely long coaxial cylinder of radii a and $b(b > a)$ respectively is filled by a homogeneous fluid of density ρ. The inner cylinder is suddenly moved with velocity v perpendicular to this axis, the outer being kept fixed. Show that the resulting impulsive pressure on a length l of inner cylinder is $\pi\rho a^2 l \frac{b^2+a^2}{b^2-a^2} v$. **[30 Marks]**

2003

98. A solid body of density ρ is in the shape of the solid formed by the revolution of the Cardioid $r = a(1+\cos\theta)$ about the initial line Show that the moment of inertia about the straight line through the pole and perpendicular to the initial line is $\frac{352}{105}\pi\rho a^2$. **[12 Marks]**

99. For an incompressible homogeneous fluid at the point (x, y, z) the velocity distribution is given by $u = -\frac{c^2 y}{r^2},\ v = \frac{c^2 x}{r^2},\ w = 0;$ where r denotes the distance from z-axis. Show that it is a possible motion and determine the surface which is orthogonal to stream line. **[12 Marks]**

100. A fine circular tube, radius c, lies on a smooth horizontal plane and contains two equal particles connected by an elastic string in the tube, the natural length of which is equal to half the circumference. The particles are in contact and fastened together, the string being stretched the tube. If the particles become disunited, prove that the velocity to the tube when the string has regained its natural length is $\left\{\frac{2\pi\lambda mc}{M(M+2m)}\right\}^{\frac{1}{2}}$ where M, m the masses of the tube and each particle respectively, and is λ the modulus of elasticity. **[30 Marks]**

101. Two sources, each of strength m are placed at the points $(-a,0)$ and $(a,0)$ and a sink of strength $2m$ is placed at the origin. Show that the stream lines are the curves $(x^2+y^2)^2 = a^2(x^2-y^2+\lambda xy)$, where λ is a variable parameter. Also show that fluid speed at any point is $\frac{2ma^2}{r_1 r_2 r_3}$ where r_1, r_2 and r_3 are respectively the distance of the point from the sources and sink. **[15 Marks]**

102. An infinite mass of fluid is acted upon by a force $\mu r^{\frac{-3}{2}}$ per unit mass directed the origin. If initially the fluid is at a rest and there is a cavity in the form of a sphere $r = c$ in it. Show that the cavity will be filled up after an interval of time $\left\{\frac{2}{5\mu}\right\}^{\frac{1}{2}} c^{\frac{5}{4}}$. **[15 Marks]**

2002

103. Find the moment of inertia of a circular wire about (i) a diameter and (ii) a line through the centre and perpendicular to its plane. **[12 Marks]**

104. Show that the velocity potential $\phi = \frac{1}{2}a\left(x^2+y^2-2z^2\right)$ satisfies the Laplace equation, and determine the stream lines. **[12 Marks]**

105. A thin circular disc of mass M and radius a can turn freely about a thin axis OA which is perpendicular to its plane and passes through a point O of its circumference. The axis OA is compelled to move in a horizontal plane with angular velocity w about its end A. Show that the inclination ϕ to the vertical of the radius of the disc through O is $\cos{-1}\left(\frac{g}{aw^2}\right)$ unless $w^2 < \frac{g}{a}$ and then θ is zero. **[30 Marks]**

106. Show that: $u=\frac{-2xyz}{\left(x^2+y^2\right)^2}, v=\frac{\left(x^2-y^2\right)z}{\left(x^2+y^2\right)^2}, w=\frac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational? **[15 Marks]**

107. Prove that $\left(v\nabla^2-\frac{\partial}{\partial t}\right)\nabla^2\psi=\frac{\partial\left(\psi,\nabla^2\psi\right)}{\partial(x,y)}$ where V is the kinematic viscosity of the fluid and ψ is the stream function for a two-dimensional motion of a viscous fluid. **[15 Marks]**

2001

108. Determine the moment of inertia of a uniform hemisphere about its axis of symmetry and about an axis perpendicular to the axis of symmetry and though centre of the base. **[12 Marks]**

109. If the velocity distribution of an incompressible fluid at the point (x,y,z) is given by $\left(\frac{3xz}{r^5},\frac{3yz}{r^5},\frac{kz^2-r^2}{r^5},\right)$ then determine the parameter k such that it is a possible motion. Hence find its velocity potential. **[12 Marks]**

110. Find the equation of motion for a particular of mass m which is constrained to move on the surface of a cone of semi-vertical angle α and which is subjected to a gravitational force. **[30 Marks]**

111. Show that the velocity distribution in axial flow of viscous incompressible fluid along a pipe of annular cross-section radii $r_1<r_2$, is given by $\omega(r)=\frac{1}{4\mu}\frac{dp}{dz}\left\{r^2-r_1^2+\frac{r_2^2-r_1^2}{\log\left(\frac{r_2}{r_1}\right)}\log\left(\frac{r}{r_2}\right)\right\}$. **[30 Marks]**

2000

112. Find the moment of inertia of an elliptic area about a line CP inclined at θ to the major axis and about a tangent parallel to CP where C is the centre of the ellipse. **[12 Marks]**

113. Determine the stream line and the path lines of the particle hen the components of the velocity field are given by $u=\frac{x}{1+t}, v=\frac{y}{2+t}$ and $w\frac{z}{3+t}$ Also state the condition for which the stream lines are identical with the path lines. **[12 Marks]**

114. A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. show that he gets to the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$ where a is the length of the plank. **[30 Marks]**

115. Define irrotational and rotational flows giving an example for each show that
$u=\frac{-2xyz}{\left(x^2+y^2\right)^2}, v=\frac{\left(x^2-y^2\right)z}{\left(x^2+y^2\right)^2}, w=\frac{y}{\left(x^2+y^2\right)}$ .are the velocity component of a possible liquid motion
Examine this for irrotational motion **[30 Marks]**

1999

116. A particle of given mass m moves in space with the Lagrangian $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-V+\dot{x}A+\dot{y}B+\dot{z}C$ where V, A, B, C are given function of x, y, z. show that the equations of motion are $m\ddot{x}=-\frac{\partial V}{\partial x}+\dot{y}\left(\frac{\partial B}{\partial x}-\frac{\partial A}{\partial y}\right)-\dot{z}\left(\frac{\partial A}{\partial z}-\frac{\partial C}{\partial x}\right)$ and two similar equations for y and z. Find also the Hamiltonian H in terms of generalized momenta. **[20 Marks]**

117. A wheel consists of a thin rim of mass M and n evenly placed spokes each of mass m, which may be considered as thin rods terminating at the centre of the wheel If the wheel is rolling with linear velocity v, express its kinetic energy in terms of M, m, n, v. With what acceleration will it roll down a rough inclined plane of inclination α ? **[20 Marks]**

118. Find the moment of inertia of a solid hemisphere about a diameter of its plane base. A solid hemisphere is held with its base against a smooth vertical wall and its lowest point on the smooth floor. The hemisphere is released. Find the initial reactions on the wall and the floor **[20 Marks]**

119. Drive the equation of continuity for a fluid in which there are no sources or sinks. Liquid flow through a pipe whose surface is the surface of revolution of the curve $y=a+kx^2/\text{a}$ about the x-axis $\left(-a\le x\le a\right)$ If the liquid enters at the end $x=-a$ of the pipe with velocity V show that the time take by a liquid particle to traverse the entire length of the pipe from $x=-a$ to $x=+a$ a is $\left\{\frac{2a}{V\left(1+k\right)^2}\right\}\left(1+\frac{2}{3}k+\frac{1}{5}k^2\right)$ (Assume that K is so small that the flow remain appreciably one-dimensional throughout.) **[20 Marks]**

120. A spherical globule of gas initially of radius $R_\circ$ and at pressure P_0 expands in an infinite mass of water of density p in which the pressure at infinity is zero. The gas is initially at rest and its pressure p and volume v are governed by the equation $pv^{4/3}=\text{constant}.$ Prove that the gas double its radius i time $\frac{28R_0}{15}\left(\frac{2\rho}{p_0}\right)^{\frac{1}{2}}$. **[20 Marks]**

121. Two sources, each of strength m are placed at the points $(-a,0)$ and $(a,0)$ and a sink of strength $2m$ is placed at the origin. Show that the stream lines are the curves $(x^2+y^2)^2=a^2(x^2-y^2+\lambda xy)$, where λ is a variable parameter. **[20 Marks]**

1998

122. Two particles in a plane are connected by a rod of constant length and are constrained to move in such a manner that the velocity of the middle of the rod is in the direction of the rod write down the equations of the constants. Is the system holenomic or non holenomic? Give reason for your answer. **[20 Marks]**

123. Using Lagrange equations obtain the differential equations of motion of a free particle in spherical polar coordinates. **[20 Marks]**

124. A rod of length 2a is suspended by a string of length l attached to one end; if the string and rod revolve about the vertical with uniform angular velocity ω, and their inclinations to the vertical be α and β respectively, show that $\omega^2=\frac{3g\tan\beta}{3l\sin\alpha+4a\sin\beta}$. **[20 Marks]**

125. A particle of mass m is fixed to a point P of the rim of a uniform circular disc of centre σ mass m and radius a. The disc is held, its plane vertical its lowest point in contact with a perfectly rough horizontal table and with OP inclined at 60° to the upward vertical and is then released. If the subsequent motion continues in the same vertical plane show that, when OP makes an angle θ with the upward vertical.

$a(7+4\cos\theta)\theta^2 = 2g(1-2\cos\theta)$. Show that when OP is first horizontal, the acceleration of σ is $\frac{18}{49}g$.

[30 Marks]

126. Three equal uniform rods AB, BC, CD each of mass m and length 2a are at rest in a straight line smoothly jointed at B and C .A blow J is given to the middle rod at a distance x from its centre σ in a direction perpendicular to it; show that the initial velocity of σ is $\frac{2J}{3m}$ and that the initial angular velocities of the are $\frac{5a+9x}{10ma^2}J, \frac{6x}{5ma^2}J, \frac{5a-9x}{10ma^2}J$ **[30 Marks]**

127. Show that a fluid of constant density can have a velocity $\vec{q}$ given by $\vec{q} = \left[\frac{-2xyz}{(x^2+y^2)^2}, \frac{(x^2-y^2)z}{(x^2+y^2)^2}, \frac{y}{x^2+y^2}\right]$.

[20 Marks]

128. Steam is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d ; If v and the corresponding velocities of the steam and if the motion be supposed to be that of divergence from the vertex of the cone, prove that $\frac{v}{V} = \left(\frac{D}{d}\right)^2 e^{\frac{v^2-V^2}{2k}}$ where k the pressure is divided by the density, and supposed constant. **[20 Marks]**

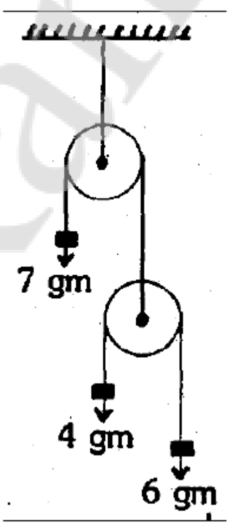
129. Between two fixed boundaries $\theta = \frac{\pi}{4}$ and $\theta = -\frac{\pi}{4}$ there is a two-dimensional liquid motion due to a source of strength m at the point $(r = a, \theta = 0)$ and an equal sink at the point $(r = b, \theta = 0)$. Show that the stream functions is $-m\tan^{-1}\left(\frac{r^4(a^4-b^4)\sin 4\theta}{r^8 - r^4(a^4+b^4)\cos 4\theta + a^4b^4}\right)$. **[20 Marks]**

1997

130. Using Lagrange's equation obtain the differential equations for planetary motion. **[20 Marks]**

131. A circular, cylinder of radius 3 cm and weight W whose centre of gravity, G, is at a distance 1cm from the axis, rolls on a horizontal plane. If motion be just started from the position of unstable equilibrium, show that the normal reaction of the plane when G ,is in its lowest position is $W(2+k^2)(1+k^2)$,where $2k$ is the radius of gyration about an axis passing through G. **[20 Marks]**

132. A pulley system is given as shown, in the diagram. Discuss the motion of the system using Lagrange's method when the pulley wheels have negligible masses and moments of inertia and their wheels are friction less.



[20 Marks]

133. A solid homogeneous sphere is rolling on the inside of a hollow sphere the centers being always in the same vertical plane. Show that the smaller sphere will make complete revolution if, when it is in lowest position, the pressure on it is greater than $\frac{34}{7}$ times its own weight. **[30 Marks]**

134. Three equal uniform rods AB, BC, CD are smooth joined B and C and the ends A and D are fastened to smooth fixed point whose distance apart is equal to the length of the rod, the frame being t rest in the form of the square A blow j is given perpendicular to AB at its middle point and in the plane square show that $\frac{3J^2}{40m}$ where m is the energy set up is where m is the mass of each rod .find also the blows at the joined B and C. **[30 Marks]**

135. Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$ is a possible from the bounding surface of a liquid and find expression for the normal velocity. **[20 Marks]**

136. A stream in a horizontal pipe after passing a contraction in the pipe, at which its cross-sectional area is A, is delivered at the atmospheric pressure at a place where the cross-sectional area is B. Show that if a side tube is connected with the pipe at the formal place water will be sucked up though it into pipe from a reservoir at a depth $\frac{s^2}{2g}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$ below the pipe where s is the delivery per second. **[20 Marks]**

137. Using the method of images prove that if there be a source m at the point z_0 in a fluid bounded by the lines $\theta = 0$ and $\theta = \frac{\pi}{3}$ the solution is usual notations $\phi + i\psi = -m\log[(z^3 - z_0^3)(z^3 - z_0'^3)]$ where $z_0 = x_0 + iy_0$ and $z_0' = x_0' - iy_0'$. **[20 Marks]**

1996

138. A uniform rod OA of length 2a free to turn about end 0 revolved with uniform angular velocity ω about the vertical OZ thought O and is inclined at a constant angle α to OZ show that the value of α is either zero or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$. **[20 Marks]**

139. Six equal uniform rods from a regular hexagon loosely jointed at the angular points rests on a smooth table a blow is given perpendicular to one of them at its middle point. Show that the opposite rod begins to move with one tenth of the velocity of the rod that is struck. **[20 Marks]**

140. A Cylinder of mass m radius R and moment of inertia I about its geometrical axis rolls down a hill without slipping under the action of gravity. If the velocity of the center of mass of the cylinder is initially v_0 .Find he velocity after the cylinder has dropped thought a vertical distance h. **[20 Marks]**

141. A perfectly rough circular hoop of diameter 24 cm roll on a horizontal floor with velocity $V\frac{cm}{\sec}$ toward an inelastic step of height 4cm, the plane of the hoop being vertical and perpendicular to the edge of the step. Prove that hoop can mount the step without losing contact at any stage if $2.4\sqrt{2g} > V > 2.4\sqrt{g}$. **[30 Marks]**

142. A homogeneous sphere rolls down an imperfectly rough fixed sphere, starting from rest at the highest point. If the sphere separates when the line joining their centers makes an angle with the vertical, show that the coefficient of friction μ satisfies the following equation $e^{\mu\pi/3} = \frac{3\sqrt{3} + 6\mu}{4(1 - 2\mu^2)}$. **[30 Marks]**

143. Show that the motion specified by $\vec{q} = \dfrac{-y\vec{i} + x\vec{j}}{x^2 + y^2}$ is a possible from for an incompressible fluid. Determine the stream lines. Show that the motion is irrotational and find the velocity potential. **[20 Marks]**

144. A sphere is at rest in an infinite mass of homogeneous liquid of density ρ .The pressure at infinity being $\bar{\omega}$ show that if the radius R of the sphere varies in any manner, the pressure at the surface of the sphere at any time is **[20 Marks]**

145. Find the stream function of two- dimensional motion due to two equal sources and an equal sink situated midway between them. In a region bounded by a fixed quadrantal arc and its radii deduce the motion due to a source and an equal sink situated at the end of one of the bounding radii. Show that the stream line leaving either end at an angle $\dfrac{\pi}{6}$ with the radius is $r^2 \sin\left(\dfrac{\pi}{6} + \theta\right) = a^2 \sin\left(\dfrac{\pi}{6} - \theta\right)$ where a is the radius of the quadrant. **[20 Marks]**

1995

146. How do you characterize

(i) the simplest dynamical system?

(ii) The most general dynamical system?

Show that the equations of motion $\dfrac{d}{dt}\dfrac{\partial L}{\partial q_k} - \dfrac{\partial T}{\partial q_k} = Qk, (k = 1, 2, ..., n)$ correspond to a non-conservative but sceleronomic system with n degrees of freedom where q, q_k, Q_k are respectively the generalized coordinates, the generalized velocities and the generalized forces. **[20 Marks]**

147. A solid uniform sphere has a light rod rigidly attached to it which passes through the center. The rod is so joined to a fixed vertical axis that the angle θ . between the rod and the axis may alter but the rod must turn with the axis. If the vertical axis be force to revolve constantly with uniform angular velocity, show that θ^2 is of the form $n^2(\cos\theta - \cos\beta)(\cos\alpha - \cos\theta)$ where n, α, β are certain constants. **[20 Marks]**

148. A uniform rod of length 20 cm which has one end attached to a fixed point by a light inextensible string of length $4\dfrac{1}{6}$ is performing small oscillation in a vertical plane about its position of equilibrium. Find its position at any time and the periods of principal oscillations. **[20 Marks]**

149. A carriage is placed on an inclined plane making an angle a with the horizon and rolls down without slipping between the wheels and the plane. The floor of the carriage is parallel to the plane and a perfectly rough ball is placed freely on it .Show that the acceleration of the carriage down the plane and a perfectly rough ball is placed freely on it show that the acceleration of the carriage down the plane is $\dfrac{14M + 4m' + 14m}{14M + 4m' + 21m} g \sin\alpha$ where M is the mass of the carriage excluding the wheels, m the sum of masses of the wheels which are uniform circular disc and M that of the ball which is a homogeneous solid sphere (the friction between the wheels and the axes is neglected) show that for the motion to be possible the coefficient of friction between the wheels and the plane must exceed the constant. $\dfrac{7(M + m) + 2M'}{14M + 21m + 4M'} \tan\alpha$. **[30 Marks]**

150. A sphere of radius a is projected up an inclined with velocity V and angular velocity Ω in the sense which would cause it to roll up; if $V > a\Omega$ and the coefficient of friction greater than $\dfrac{2}{7}\tan\alpha$ show that the sphere will cease to ascend at the end of a time $\dfrac{5V + 2a\Omega}{5g\sin\alpha}$ where α is the inclination of the plane . **[30 Marks]**

151. Determine the restrictions on f_1, f_2, f_3 if $f_1(t)\dfrac{x^2}{a^2} + f_2(t)\dfrac{y^2}{b^2} + f_3(t)\dfrac{z^2}{c^2} = 1$ is a possible boundary surface of a liquid. **[20 Marks]**

152. If a, b, c, d, e, f are arbitrary constants, what type of fluid motion does the velocity $(a+by-cz, d-bx+ez, f+cx-ey)$ represent? **[20 Marks]**

153. If the fluid fill the region of spaces on the positive side of x-axis, which is a rigid boundary and if there be a source $+m$ at the point $(0,a)$ and an equal sink at $(0,b)$ and if the pressure on the negative side of the boundary be the same as the pressure of the fluid at infinity, Show that the resultant pressure on the boundary is $\pi\rho m^2 \dfrac{(a-b)^2}{ab(a+b)}$, where ρ is the density of the fluid.

[20 Marks]

1994

154. What is D' Alembert's principle? An inextensible string of negligible mass hanging over a smooth page at A connects the mass m_1 on a frictionless inclined of angle θ to another mass m_2. Use D' Alembert's principle to prove that the mass will be in equilibrium if $m_2 = m_1 \sin\theta$. **[20 Marks]**

155. Two mass points of mass m_1 and m_2 are connected by a string passing thought a hole in smooth table so that rests on the table so that m_1 rests on the table surface and hangs suspended .Assuming m_2 moves only in a vertical line what are the generalized coordinates of the system? Write down Lagrange's equations of motion and obtain a first integral of the equations of motion. **[20 Marks]**

156. Twelve equal uniform rods are smoothly joined at their ends so as to form cubical framework, which is suspended from a point by string tied to one corner and kept in shape by a light string occupying the position of a vertical diagonal. Suppose that the string supporting the framework is cut, so that it falls and strikes a smooth inelastic horizontal plane. Find the impulsive reaction of the plane. **[20 Marks]**

157. A small light ring is threaded on a fixed thin horizontal wire. One end of a uniform rod of mass m and length $2a$ is freely attached to the ring. The coefficient of friction between the ring and the wire is μ the system is released from rest when the rod is horizontal and the vertical plane containing the wire. If the ring slips on the wire when the rod has turned though an angle α then prove that $\mu(10\tan^2\alpha+1) = 9\tan\alpha$.

[30 Marks]

158. A uniform rod AB held at an inclination α to the vertical with one end A in contact with a rough horizontal table. If released, then prove that the rod will commence to slide at cone if the coefficient of the friction is less than $\dfrac{3\sin\alpha\cos\alpha}{1+3\cos^2\alpha}$. **[30 Marks]**

159. The particle velocity for a fluid motion referred rectangular axes is given by $\left(A\cos\dfrac{\pi x}{2a}\cos\dfrac{\pi z}{2a}, 0, A\sin\dfrac{\pi x}{2a}\sin\dfrac{\pi z}{2a}\right)$ where $\mathrm{A,a}$ are constants. Show that this is possible motion of an incompressible fluid under no body forces in an infinite fixed right tube $-a \le x \le a,\ 0 \le z \le 2a$ Also find the pressure associated with this velocity field. **[20 Marks]**

160. Determine the stream lines and the path lines of the particles when velocity field is given by $\left(\dfrac{x}{1+t}, \dfrac{y}{1+t}, \dfrac{z}{1+t}\right)$ **[20 Marks]**

161. Between the fixed boundaries $\theta = \dfrac{\pi}{4}$ and $\theta = -\dfrac{\pi}{4}$, there is a two-dimensional liquid motion due to a source of strength m at the point $r = a, \theta = 0$ and an equal sink at the point $r = b, q = 0$. Use the method of images to show that the stream function is $-m\tan^{-1}\left\{\dfrac{r^4(a^4-b^4)\sin 4\theta}{r^8 - r^4(a^4+b^4)\cos 4\theta + a^4b^4}\right\}$,Show also that the velocity at (r,θ) is $\dfrac{4m(a^4-b^4)r^3}{(r^8-2a^4r^4\cos 4\theta+a^8)^{1/2}(r^8-2b^4r^4\cos 4\theta+b^8)^{1/2}}$ **[20 Marks]**

1993

162. Consider the two dynamical systems:
(i)A sphere rolling down from the top of a fixed sphere
(ii) A cylinder rolling without slipping down a rough inclined plane.
(iii) State whether (I) is rheonomic, holonomic and justify your claim
(iv)Give reasons for (II) to be classified as scleronomic holonomic Is it conservative? **[20 Marks]**

163. Find
(i)the Lagrangian
(ii) The equation of motion for the following system:
A particle is constrained to move in a plain under the influence of an attraction towards to origin proportional to the distance from it and also of a force perpendicular to the radius vector inversely proportional to the distance of the particle from the origin in anticlockwise direction. **[20 Marks]**

164. A heavy uniform rod rotating in a vertical plane fall and strikes a smooth inelastic horizontal plane. Find the impulse. **[20 Marks]**

165. The door of a railway carriage has its hinges, supposed smooth, towards the engine which start with an acceleration f. Prove that the door closed in time $\left(\frac{a^2+K^2}{2af}\right)^{1/2}\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}$ with an angular velocity $\sqrt{\frac{2af}{a^2+K^2}}$ where 2a is the breadth of the door and K its radius of gyration about a vertical axis through G, the center of mass. **[30 Marks]**

166. A solid homogeneous sphere is resting on the top another fixed sphere and roll down it. Write down the equation of smooth and find the friction when does the upper sphere leave the lower sphere if
(i) both the spheres are smooth
(ii) The upper sphere is sufficiently rough so not to slip. **[30 Marks]**

167. Show that $u=\frac{-2xyz}{\left(x^2+y^2\right)^2}, v=\frac{\left(x^2-y^2\right)z}{\left(x^2+y^2\right)^2}, w=\frac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational? **[20 Marks]**

168. Steam is rushing from a boiler though conical pipe the diameter of the ends of which are D and d ;If V and v be the corresponding velocities of the steam and if the motion be supposed to be that of divergence from the vertex of the cone, prove that $\frac{v}{V}=\frac{D^2}{d^2}e(v^2-V^2)/2K$ where K is the pressure divided by the density and supposed constant. **[20 Marks]**

169. Prove that for liquid circulating irrotationally in part of the plane between two non- intersecting circle the curve of constant velocity are Cassini's ovals. **[20 Marks]**

1992

170. Classify each of the following dynamical systems according as they are
   a. scleronomic or rheonomic
   b. holoniomic or non-holonomic
   c. conservative or non-conservative
   [I] a horizontal cylinder of radius a rolling inside a perfectly rough hollow horizontal cylinder of radius $b>a$

[II] A particle constrained to move along a line under the influence of a force which is inversely proportional to the square of its distance from a fixed point and a damping force proportional to the square of its distance from a fixed point and a damping force proportional to the square of the instantaneous speed.

[III] a particle moving on a very long frictionless wire which rotates with constant angular speed about a horizontal axis.

[10 Marks]

171. When the Lagrangian function has the form $L = q_k q_k - \sqrt{1-q^2{}_k}$ .Show that generalized acceleration is zero. **[20 Marks]**

172. The end of a uniform rod AB of length $2a\cos 15^o$ and weight W are constrained to slide on a smooth circular wire of radius a fixed with its plane vertical. The end A is connected by an elastic string of natural length a and modulus of elasticity $W/2$ to the highest point of the wire. If θ is the angle which the perpendicular bisector of the rod makes with the downward vertical, show that the potential energy V is given by

$$V = -\frac{W_a}{2}\left\{\cos\left(\theta - 75^o\right) + 2\cos\frac{1}{2}\left(\theta + 75^o\right)\right\} + \text{constant}$$

verify that $\theta = 25°$ defines a position of equilibrium and investigate its stability. **[30 Marks]**

173. A uniform rod of length 2a which has one end attached to a fixed point by a light inextensible sting of length $\frac{5a}{12}$ performing small oscillations in a vertical plane about its position of equilibrium. Find the position at any time show that the periods of its principal oscillations are $2\pi\sqrt{\frac{5a}{3g}}$ and $\pi\sqrt{\frac{a}{3g}}$. **[30 Marks]**

174. A uniform circular disc of radius a and mass m rolls down a rough inclined plane without sliding. Show that the center of the disc moves with contact acceleration $2/3g\sin\alpha$ and the coefficient of friction $\mu > 1/3\tan$ where α is the inclination of the plane. **[30 Marks]**

175. Show that the variable ellipsoid $\frac{x^2}{a^2k^2t^4} + kt^2\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$ is a possible form the bounded surface of a liquid motion at any time t. **[20 Marks]**

176. Find the lines of flow in the 2- dimensional fluid motion given by $\phi + i\psi = -\frac{1}{2}n(x+iy)^2 e^{2int}$. Prove or verify that the paths of the particles of the fluid (in polar coordinates) may be obtained by eliminating t from the equations $r\cos(nt+\theta) - x_0 = r\sin(nt+\theta) - y_0 = nt(x_0 - y_0)$. **[20 Marks]**

177. A source of strength m and a vortex of strength k are placed at the origin of the 2- dimensional motion of unbounded liquid prove that the pressure at infinity exceeds that pressure at distance r from the origin by $\frac{1}{2} - \frac{\left(m^2+k^2\right)}{r^2}p$. **[20 Marks]**