RAMANASRI
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Modern Algebra

IFoS (IFS) Previous Year Questions (PYQ) from 2025 to 2009

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2025

1. If H and K are finite subgroups of a group and $HK = \{hk : h \in H,\ k \in K\}$, prove that $|HK| = \frac{|H||K|}{|H \cap K|}$. **[8 Marks]**

2. If p is an odd prime, prove that there is no group that has exactly p elements of order p. **[10 Marks]**

3. Show that in the ring $\mathbb{Z} \times \mathbb{Z}$, (i) the ideal $S = \{(a, 0) : a \in \mathbb{Z}\}$ is a prime ideal but not maximal and (ii) the ideal $T = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} : 3 \text{ divides } n\}$ is a maximal ideal. **[10+5 Marks]**

4. Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0 \in \mathbb{Z}[x]$, and suppose that p is prime such that $p \nmid a_n$, $p \mid a_{n-1}, \ldots, p \mid a_0$ and $p^2 \nmid a_0$. Prove that $f(x)$ is irreducible over $\mathbb{Q}$. **[10 Marks]**

2024

5. Let R be the ring of $n \times n$ matrices over $\mathbb{R}$. Show that R has only two ideals, namely $\{0\}$ and R. **[8 Marks]**

6. If G is a group of even order, show that there exists an element a other than the identity such that $a^2 = e$. Also prove that an ideal S of $\mathbb{Z}$ is maximal if S is generated by a prime integer. **[5+5 Marks]**

7. Prove that in a Unique Factorization Domain R, an element is prime if and only if it is irreducible. **[15 Marks]**

8. Let G and H be finite groups such that $\gcd(|G|, |H|) = 1$. Show that the trivial homomorphism is the only homomorphism from G into H. **[10 Marks]**

2023

9. Prove that a subgroup of a cyclic group is cyclic. Let G be a cyclic group with generator a. If the order of G is infinite, prove that G is isomorphic to $(\mathbb{Z}, +)$. **[8 Marks]**

10. Prove that every group is isomorphic to a group of permutations. Let $A = \{1, 2, 3\}$ and let S_3 denote the symmetric group on 3 elements. Then is S_3 an abelian or non-abelian group? **[5+5 Marks]**

11. If N is a normal subgroup of a group G and if H is any subgroup of G, prove that $H \vee N = HN = NH$, where $H \vee N$ denotes the join of H and N. State the Second Isomorphism Theorem of groups and apply it to the case $G = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$, $H = \mathbb{Z} \times \mathbb{Z} \times \{0\}$ and $N = \{0\} \times \mathbb{Z} \times \mathbb{Z}$. **[8+7 Marks]**

12. Prove that the integral domain $\mathbb{Z}$ is a Unique Factorization Domain and a Euclidean Domain. **[10 Marks]**

2022

13. Let F be a finite field of characteristic p, where p is prime. Show that there is an injective homomorphism from $\mathbb{Z}_p$ into F. Also show that the number of elements in F is p^n, for some positive integer n. **[8 Marks]**

14. Find all the Sylow p-subgroups of S_4 and show that none of them is normal. **[10 Marks]**

15. Let P be a Sylow p-subgroup of a group G and H any p-subgroup of G such that $HP = PH$. Show that $H \subseteq P$. Also show that every group of order 15 is cyclic. **[7+8 Marks]**

16. Prove that $R[x]$ is a principal ideal domain if and only if R is a field. **[10 Marks]**

2021

17. Let G be a finite commutative group. Let $n \in \mathbb{Z}$ be such that n and the order of G are relatively prime. Show that the function $\varphi : G \to G$ defined by $\varphi(a) = a^n$, for all $a \in G$, is an isomorphism of G onto G. **[8 Marks]**

18. Prove that every group is isomorphic to a permutation group. **[10 Marks]**

19. Let R be a non-zero commutative ring with unity. If every ideal of R is prime, prove that R is a field. Also, let R be a commutative ring with unity such that $a^2 = a$, for all $a \in R$. If I is any prime ideal of R, find all the elements of R/I. **[8+7 Marks]**

20. Show that an element x in a Euclidean domain is a unit if and only if $d(x) = d(1)$, where the notations have their usual meanings. **[10 Marks]**