UPSC MATHS OPTIONAL COACHING

Numerical Analysis & Computer Programming

Previous year Questions

from 2025 to 1992

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2025

1. Solve the following system of linear equations by Gauss-Seidel method:

$$10x + 2y + z = 9$$
$$2x + 20y - z = -44$$
$$-2x + 3y + 10z = 22$$

[10 Marks]

2. (i) Convert the number $(3479)_{10}$ into binary system and the number $(7AE.9F)_{16}$ into decimal system.
(ii) Determine the truth table for the Boolean function $F(x, y, z) = (x + y + z')(x' + y')$. Also derive the full disjunctive normal form of $F(x, y, z)$ from the truth table. **[10 Marks]**

3. Simplify the Boolean function $F(x, y, z) = xyz + x'yz + xy'z + xyz'$ and draw the corresponding GATE network. **[15 Marks]**

4. Find the unique polynomial of degree 2 or less which fits the following data:

x	0	1	3
$f(x)$	1	3	55

Also obtain the bound on the truncation error. **[15 Marks]**

5. Find the constant p and error term for the quadrature formula

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}(f_0 + f_1) + ph^2(f_0' - f_1'),$$

where $x_0 + h = x_1$, $f_0 = f(x_0)$, $f_1 = f(x_1)$, and prime denotes derivative with respect to x. Hence deduce the composite rule for integrating $\int_a^b f(x)\,dx$, where $a = x_0 < x_1 < \cdots < x_N = b$. **[15 Marks]**

2024

6. Solve the following system of linear equations by Gauss-Jordan method:

$$2x + 3y - z = 5$$
$$4x + 4y - 3z = 3$$
$$2x - 3y + 2z = 2$$

[10 Marks]

7. (i) Determine the decimal equivalent in sign magnitude form of $(8D)_{16}$ and $(FF)_{16}$.
(ii) Determine the decimal equivalent of $(9B2.1A)_{16}$. **[10 Marks]**

8. Draw the logical circuit for the Boolean expression $Y = ABC' + BC' + A'B$. Also, obtain the output Y for the three input bit sequences:

$$A = 10001111, \quad B = 00111100, \quad C = 11000100.$$

[15 Marks]

9. Integrate $f(x) = 5x^3 - 3x^2 + 2x + 1$ from $x = -2$ to $x = 4$ using (i) Simpson's $\frac{3}{8}$ rule with width $h = 1$, and (ii) Trapezoidal rule with width $h = 1$. **[15 Marks]**

10. Using Newton's forward difference formula for interpolation, estimate the value of $f(2.5)$ from the following data:

x	1	2	3	4	5	6
$f(x)$	0	1	8	27	64	125

[15 Marks]

2023

11. Given $\dfrac{dy}{dx} = \dfrac{y^2 - x}{y^2 + x}$, with initial condition $y = 1$ at $x = 0$. Find the value of y for $x = 0.4$ by Euler's method, correct to 4 decimal places, taking step length $h = 0.1$. **[10 Marks]**

12. Evaluate, using binary arithmetic, the following numbers in their given system: (i) $(634.235)_8 - (132.223)_8$; (ii) $(7AB.432)_{16} - (5CA.D61)_{16}$. **[10 Marks]**

13. Solve the system of linear equations

$$7x_1 - x_2 + 2x_3 = 11$$
$$2x_1 + 8x_2 - x_3 = 9$$
$$x_1 - 2x_2 + 9x_3 = 7$$

correct up to 4 significant figures by the Gauss-Seidel iterative method. Take initially guessed solution as $x_1 = x_2 = x_3 = 0$. **[15 Marks]**

14. (i) Find the conjunctive normal form (CNF) of the Boolean function $f(x, y, z, t) = x \cdot y \cdot z + x' \cdot y \cdot (t + z')$.
(ii) Express $f(x, y, z) = x + (x' \cdot y' + x' \cdot z') + z$ in disjunctive normal form (DNF) and construct the truth table for the function. **[15 Marks]**

15. Compute a root of the equation $\log_{10}(2x + 1) - x^2 + 3 = 0$, in the interval $[0, 3]$, by Regula-Falsi method, correct to 6 decimal places. **[15 Marks]**

2022

16. Solve, by Gauss elimination method, the system of equations

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + y + 3z = 14$$

[10 Marks]

17. (i) Convert the number $(1093.21875)_{10}$ into octal and the number $(1693.0628)_{10}$ into hexadecimal systems.
(ii) Express the Boolean function $F(x, y, z) = xy + x'z$ in product of maxterms form. **[10 Marks]**

18. Find a combinatorial circuit corresponding to the Boolean function $f(x, y, z) = [x \cdot (y' + z)] + y$ and write the input/output table for the circuit. **[15 Marks]**

19. The velocity of a train which starts from rest is given in the following table, the time being reckoned in minutes from the start and the velocity in km/hour:

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32	17.6	8	3.2	0

Using Simpson's $\frac{1}{3}$ rule, estimate approximately in km the total distance run in 20 minutes. **[15 Marks]**

20. Using Runge-Kutta method of fourth order, solve the differential equation $\dfrac{dy}{dx} = x + y^2$ with $y(0) = 1$, $x = 0.2$. Use four decimal places for calculation and step length 0.1. **[15 Marks]**

2021

21. Find a positive root of the equation $3x = 1 + \cos x$ by a numerical technique using initial values 0 and $\frac{\pi}{2}$ and further improve the result using Newton-Raphson method correct to 8 significant figures.
[10 Marks]

22. (i) Convert $(3798.3875)_{10}$ into octal and hexadecimal equivalents.
(ii) Obtain the principal conjunctive normal form of $(\neg P \to R) \wedge (Q \Leftrightarrow P)$. **[10 Marks]**

23. Obtain the Boolean function $F(x, y, z)$ based on the table given below. Then simplify $F(x, y, z)$ and draw the corresponding GATE network:

x	y	z	F
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

[15 Marks]

24. Solve the system of equations

$$3x_1 + 9x_2 - 2x_3 = 11$$
$$4x_1 + 2x_2 + 13x_3 = 24$$
$$4x_1 - 2x_2 + x_3 = -8$$

correct up to 4 significant figures by using Gauss-Seidel method after verifying whether the method is applicable in your transformed form of the system. **[15 Marks]**

25. Derive Newton's backward difference interpolation formula and also do error analysis. **[15 Marks]**

2020

26. Show that the equation $f(x) \equiv \cos\frac{\pi(x+1)}{8} + 0.148x - 0.9062 = 0$ has one root in the interval $(-1, 0)$ and one in $(0,1)$. Calculate the negative root correct to four decimal places using Newton-Raphson Method.
[10 Marks]

27. Let $g(w,x,y,z) = (w + x + y)(x + \overline{y} + z)(w + \overline{y})$ be a Boolean function. Obtain the conjunctive normal form for $g(w,x,y,z)$. Also express $g(w,x,y,z)$ as product of maxterms. **[10 Marks]**

28. For the solution of system of equations:

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

Set up the Gauss-Seidel iterative scheme and iterate three times starting with the initial vector $X^{(0)} = 0$.
Also find the exact solutions and compare with the iterated solutions. **[15 Marks]**

29. Find a quadrature formula $\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$ which is exact for polynomials of highest possible degree. Then use the formula to evaluate $\int_0^1 \frac{dx}{\sqrt{x - x^3}}$ (correct up to three decimal places).

[20 Marks]

30. Write the three-point Lagrangian interpolating polynomial relative to the points $x_0, x_0+\varepsilon$ and x_1. Then by taking the limit $\varepsilon \to 0$, establish the relation

$$f(x) = \frac{(x_1 - x)(x + x_1 - 2x_0)}{(x_1 - x_0)^2} f(x_0) + \frac{(x - x_0)(x_1 - x)}{(x_1 - x_0)} f'(x_0) + \frac{(x - x_0)^2}{(x_1 - x_0)} f(x_1) + E(x)$$

where $E(x) = \frac{1}{6}(x_1 - x_0)^2 (x - x_1) f'''(\xi)$ is the error function and

$\min(x_0, x_0 + \varepsilon, x_1) < \xi < \max(x_0, x_0 + \varepsilon, x_1)$. **[15 Marks]**

2019

31. Apply Newton-Raphson method, to find real root of transcendental equation, $x\log_{10} x = 1.2$ correct to three decimal places. **[10 Marks]**

32. Using Runge-Kutta method of forth order to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$. Use four decimal places for calculation and step length 0.2 **[10 Marks]**

33. Draw a flow chart and write a basic algorithm for (in FORTRAN/C/C++) for evaluating $y = \int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule **[10 Marks]**

34. Find the equivalent numbers given in a specified number to the system mentioned against them:
(i) Integer 524 in binary system.
(ii) 101010110101. 101101011 to octal system.
(iii) decimal number 5280 to hexadecimal system.

35. (iv) Find the unknown number $(1101.101)_8 \to (?)_{10}$. **[15 Marks]**

36. Apply Gauss-Seidel iteration method to solve the following system of equations: $2x + y - 2z = 17$ $3x + 20y - z = 18$ $2x - 3y + 20z = 25$, correct to three decimal places. **[15 Marks]**

37. Given the Boolean expression. $X = AB + ABC + A\overline{B}\overline{C} + \overline{A}\overline{C}$
(i) Draw the logical diagram for the expression.
(ii) Minimize the expression.
(iii) Draw the logical diagram for the reduced expression. **[15 Marks]**

2018

38. Using Newton's forward difference formula find the lowest degree polynomial u_x when it is given that $u_1 = 1, u_2 = 9, u_3 = 25, u_4 = 55$ and $u_2 = 105$. **[10 Marks]**

39.

40. Starting from rest in the beginning, the speed (in km/h) of a train at different times (in minutes) is given by the below table:

Time(Minutes)	2	4	6	8	10	12	14	16	18	20
Speed(Km/h)	10	18	25	29	32	20	11	5	2	8.5

Using Simpsons' $\frac{1}{3}rd$ rule, Find the approximate distance travelled (in km) in 20 minutes from the beginning. **[10 Marks]**

41. Write down the basic algorithm for solving the equation $xe^{x}-1=0$ by bisection method, correct to 4 decimal places. **[10 Marks]**

42. Find the equivalent of numbers given in a specified number system to the system mentioned against them. **[15 Marks]**

(i) $(111011\cdot101)_2$ to decimal system

(ii) $(1000111110000\cdot00101100)_2$ to hexadecimal system

(iii) $(C4F2)_{16}$ to decimal system

(iv) $(418)_{10}$ to binary system

43. Simplify the Boolean expression: $(a+b).(\bar{b}+c)+b.(\bar{a}+\bar{c})$ By using the laws of Boolean algebra. From its truth table write it in min-terms normal form. **[15 Marks]**

44. Find the values of the constants a,b,c such that the quadrature formula

$\int_o^h f(x)dx = h\left[af(o)+bf\left(\frac{h}{3}\right)+cf(h)\right]$ is exact for polynomials of as high degree as possible, and hence find the order of the truncation error. **[15 Marks]**

2017

45. Explain the main steps of the Gauss-Jordan method and apply this method to find the inverse of the matrix $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$. **[10 Marks]**

46. Write the Boolean expression $z(y+z)(x+y+z)$ in the simplest form using Boolean postulate rules. Mention the rules used during simplification. Verify your result by constructing the truth table for the given expression and for its simplest form. **[10 Marks]**

47. For given equidistant values u_{-1}, u_0, u_1 and u_2 a values are interpolated by Lagrange's formula. Show that it may be written in the form $u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_0$, where $x+y=1$. **[15 Marks]**

48. Derive the formula $\int_a^b ydx = \frac{3h}{8}[(y_0+y_n)+3(y_1+y_2+y_4+y_5+...+y_{n-1})+2(y_3+y_6+y_{n-3})]$. Is there any restriction on n? State that condition. What is the error bounded in the case of Simpson's $\frac{3}{8}$ rule? **[20 Marks]**

49. Write an algorithm in the form of a flow chart for Newton-Raphson method. Describe the cases of failure of this method. **[15 Marks]**

2016

50. Convert the following decimal numbers to univalent binary and hexadecimal numbers:

[i] 4096 [ii] 0.4375 [iii] 2048.0625 **[10 marks]**

51. Let $f(x)=e^{2x}\cos 3x$ for $x\in[0,1]$. Estimate the value of $f(0.5)$ Using Lagrange interpolating polynomial of degree 3 over the nodes $x=0, x=0.3, x=0.6$ and $x=1$. Also compute the error bound over the interval $[0,1]$ and the actual error $E(0.5)$ **[20 marks]**

52. For an integral $\int_{-1}^{1} f(x)dx$ show that the two-point Gauss quadrature rule is given by

$\int_{-1}^{1} f(x)dx = f\left(\frac{1}{\sqrt{3}}\right)+f\left(-\frac{1}{\sqrt{3}}\right)$ using this rule estimate $\int_{2}^{4} 2xe^{x}dx$ **[15 marks]**

53. Let A, B, C be Boolean variable denote complement $A, A+B$ of is an expression for $A\ OR\ B$ and $B.A$ is an expression for $AANDB$. Then simplify the following expression and draw a block diagram of the simplified expression using AND and OR gates.

$A.(A+BC).(\overline{A}+B+C).(A+\overline{B}+C).(A+B+\overline{C})$. **[15 marks]**

2015

54. Find the principal **[**or canonical**]** disjunctive normal form in three variables p,q,r for the Boolean expression $\left((p\wedge q)\to r\right)\vee\left((p\wedge q)\to -r\right)$. Is the given Boolean expression a contradiction or a tautology? **[10 Marks]**

55. Find the Lagrange interpolating polynomial that fits the following data:

x	-1	2	3	4
$f(x)$	-1	11	31	69

Find $f(1.5)$ **[20 Marks]**

56. Solve the initial value problem $\frac{dy}{dx}=x(y-x),\ y(2)=3$ in the interval $[2,\ 2.4]$ using the Runge-Kutta fourth-order method with step size $h=0.2$ **[15 Marks]**

57. Find the solution of the system

$$10x_1-2x_2-x_3-x_4=3$$
$$-2x_1+10x_2-x_3-x_4=15$$
$$-x_1-x_2+10x_3-2x_4=27$$
$$-x_1-x_2-2x_3+10x_4=-9$$

using Gauss-Seidel method (make four iterations) **[15 Marks]**

2014

58. Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^{x}=0$ correct up to four decimal places. **[10 Marks]**

59. Use five subintervals to integrate $\int_{0}^{1}\frac{dx}{1+x^2}$ using trapezoidal rule. **[10 Marks]**

60. Use only AND and OR logic gates to construct a logic circuit for the Boolean expression $z=xy+uv$ **[10 Marks]**

61. Solve the system of equations

$$2x_1-x_2=7$$
$$-x_1+2x_2-x_3=1$$

$-x_2 + 2x_3 = 1$

using Gauss-Seidel iteration method [perform three iterations] **[15 Marks]**

62. Use Runge-Kutta formula of fourth order to find the value of y at $x = 0.8$, where $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$. Take the step length $h = 0.2$ **[20 Marks]**

63. Draw a flowchart for Simpson's one-third rule. **[15 Marks]**

64. For any Boolean variables x and y, show that $x + xy = x$. **[15 Marks]**

2013

65. In an examination, the number of students who obtained marks between certain limits were given in the following table:

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

Using Newton's forward interpolation formula, find the number of students whose marks lie between 45 and 50. **[10 Marks]**

66. Develop an algorithm for Newton-Raphson method to solve $f(x) = 0$ starting with initial iterate x_0, n be the number of iterations allowed, epsilon be the prescribed relative error and delta be the prescribed lower bound for $f'(x)$ **[20 Marks]**

67. Use Euler's method with step size $h = 0.15$ to compute the approximate value of $y(0.6)$, correct up to five decimal places from the initial value problem. $y' = x(y+x) - 1,\ y(0) = 2$ **[15 Marks]**

68. The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using composite Simpson's $\frac{1}{3}$ rule.

[15 Marks]

2012

69. Use Newton-Raphson method to find the real root of the equation $3x = \cos x + 1$ correct to four decimal places **[12 Marks]**

70. Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ in the interval $[a, b]$ for n number of discrete points, where the initial value is $y(a) = \alpha$, using Euler's method.

[15 Marks]

71. Solve the following system of simultaneous equations, using Gauss-Seidel iterative method :

$$3x + 20y - z = -18$$
$$20x + y - 2z = 17$$
$$2x - 3y + 20z = 25$$

[20 Marks]

72. Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data:

x:	0.1	0.2	0.3	0.4
y:	0.9975	0.9900	0.9776	0.9604

[20 Marks]

73. In a certain examination, a candidate has to appear for one major & two major subjects .The rules for declaration of results are marks for major are denoted by M_1 and for minors by M_2 and M_3.If the candidate obtains 75% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class with distinction. If the candidate obtains 60% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class. If the candidate obtains 50% or above in major, 40% or above in each of the two minors and an average of 50% or above in all the three subjects put together, the candidate is declared to have passed the examination in second class. All those candidates, who have obtained 50% and above in major and 40% or above in minor, are declared to have passed the examination. If the candidate obtains less than 50% in major or less than 40% in anyone of the two minors, the candidate is declared to have failed in the examinations. Draw a flow chart to declare the results for the above. **[20 Marks]**

2011

74. Calculate $\int_2^{10} \frac{dx}{1+x}$ **[**up to 3 places of decimal**]** by dividing the range into 8 equal parts by Simpson's $\frac{1}{3}^{rd}$ rule. **[12 Marks]**

75. [i] Compute $(3205)_{10}$ to the base 8.

[ii] Let A be an arbitrary but fixed Boolean algebra with operations $\wedge, \vee$ and $'$ and the zero and the unit element denoted by 0 and 1 respectively. Let $x, y, z ...$ be elements of A. If $x, y \in A$ be such that $x \wedge y = 0$ and $x \vee y = 1$ then prove that $y = x' ...$ **[12 Marks]**

76. A solid of revolution is formed by rotating about the x- axis, the area between the x- axis, the line $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates:

x	0.00	0.25	0.50	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Find the volume of the solid. **[20 Marks]**

77. Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit:

x	y	z	$f(x,y,z)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

[20 Marks]

78. Draw a flow chart for Lagrange's interpolation formula. **[20 Marks]**

2010

79. Find the positive root of the equation $10xe^{-x^2} - 1 = 0$ correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. **[12 Marks]**

80. [i] Suppose a computer spends 60 per cent of its time handling a particular type of computation when running a given program and its manufacturers make a change that improves its performance on that type of computation by a factor of 10. If the program takes 100 sec to execute, what will its execution time be after the change?

[ii] If $A \oplus B = AB' + A'B$, find the value of $x \oplus y \oplus z$. **[6+6=12 Marks]**

81. Given the system of equations

$$2x + 3y = 1$$
$$2x + 4y + z = 2$$
$$2y + 6z + Aw = 4$$
$$4z + Bw = C$$

State the solvability and uniqueness conditions for the system. Give the solution when it exists.

[20 Marks]

82. Find the value of the integral $\int_1^5 \log_{10} x \, dx$ by using Simpson's $\frac{1}{3}$ rd rule correct up to 4 decimal places. Take 8 subintervals in your computation. **[20 Marks]**

83. [i] Find the hexadecimal equivalent of the decimal number $(587632)_{10}$

[ii] For the given set of data points $(x_1, f(x_1), (x_2, f(x_2), \ldots (x_n, f(x_n)$ write an algorithm to find the value of $f(x)$ by using Lagrange's interpolation formula

[iii] Using Boolean algebra, simplify the following expressions

[a] $a + a'b + a'b'c + a'b'c'd + \ldots$

[b] $x'y'z + yz + xz$ where x' represents the complement of x **[5+10+5=15 Marks]**

2009

84. [i] The equation $x^2 + ax + b = 0$ has two real roots α and β . Show that the iterative method given by: $x_{k+1} = -\frac{(ax_k + b)}{x_k}, k = 0,1,2\ldots$ is convergent near $x = \alpha$, if $|\alpha| > |\beta|$

[ii] Find the values of two valued Boolean variables A, B, C, D by solving the following simultaneous equations:

$$\bar{A} + AB = 0$$
$$AB + AC$$
$$AB + A\bar{C} + CD = \bar{C}D$$

where $\bar{x}$ represents the complement of x **[6+6=12 Marks]**

85. [i] Realize the following expressions by using NAND gates only:

$g = (\bar{a} + \bar{b} + c)\bar{d}(\bar{a} + e)f$ where $\bar{x}$ represents the complement of x

[ii] Find the decimal equivalent of $(357.32)_8$ **[6+6=12 Marks]**

86. Develop an algorithm for Regula-Falsi method to find a root of $f(x) = 0$ starting with two initial iterates x_0 and x_1 to the root such that $\text{sign}(f(x_0)) \neq \text{sign}(f(x_1))$. Take n as the maximum number

of iterations allowed and epsilon be the prescribed error. **[30 Marks]**

87. Using Lagrange interpolation formula, calculate the value of $f(3)$ from the following table of values of x and $f(x)$:

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

[15 Marks]

88. Find the value of $y(1.2)$ using Runge-Kutta fourth order method with step size $h = 0.2$ from the initial value problem: $y' = xy,\ y(1) = 2$ **[15 Marks]**

2008

89. Find the smallest positive root of equation $xe^x - \cos x = 0$ using Regula-Falsi method. Do three iterations. **[12 Marks]**

90. State the principle of duality

(i) in Boolean algebra and give the dual of the Boolean expressions $(X+Y).(\overline{X}.\overline{Z}).(Y+Z)$ and $X\overline{X} = 0$

(ii) Represent $(\overline{A}+\overline{B}+\overline{C})(A+\overline{B}+C)(A+B+\overline{C})$ in NOR to NOR logic network.

[6+6=12 Marks]

91. [i] The following values of the function $f(x) = \sin x + \cos x$ are given:

x	10^0	20^0	30^0
$f(x)$	1.1585	1.2817	1.3360

Construct the quadratic interpolating polynomial that fits the data. Hence calculate $f\left(\frac{\pi}{12}\right)$.

Compare with exact value.

[ii] Apply Gauss-Seidel method to calculate x, y, z from the system:

$$-x - y + 6z = 42$$
$$6x - y - z = 11.33$$
$$-x + 6y - z = 32$$

with initial values $(4.67,\ 7.62,\ 9.05)$. Carry out computations for two iterations

[15+15=30 Marks]

92. Draw a flow chart for solving equation $F(x) = 0$ correct to five decimal places by Newton-Raphson method **[30 Marks]**

2007

93. Use the method of false position to find a real root of $x^3 - 5x - 7 = 0$ lying between 2 and 3 and correct to 3 places of decimals. **[12 Marks]**

94. Convert:

(i) 46655 given to be in the decimal system into one in base 6.

(ii) $(11110.01)_2$ into a number in the decimal system. **[6+6=12 Marks]**

95. [i] Find from the following table, the area bounded by the x-axis and the curve $y = f(x)$ between $x = 5.34$ and $x = 5.40$ using the trapezoidal rule:

x	5.34	5.35	5.36	5.37	5.38	5.39	5.40
$f(x)$	1.82	1.85	1.86	1.90	1.95	1.97	2.00

[15 Marks]

[ii] Apply the second order Runge-Kutta method to find an approximate value of y at $x = 0.2$ taking $h = 0.1$, given that y satisfies the differential equation and the initial condition $y' = x + y, y(0) = 1$ **[15 Marks]**

2006

96. Evaluate $I = \int_0^1 e^{-x^2} dx$ by the Simpson's rule

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2)\right] + 4f(x_3) + ... + 2f(x_{2n-2}) + 4f(x_{2n-1}) + f(x_{2n})\right]$$ with

$2n = 10, \Delta x = 0.1, x_0 = 0, x_1 = 0.1, ..., x_{10} = 1.0$ **[12 Marks]**

97. [i] Given the number 59.625 in decimal system. Write its binary equivalent.
[ii] Given the number 3898 in decimal system. Write its equivalent in system base 8.
[6+6=12 Marks]

98. If Q is a polynomial with simple roots $\alpha_1, \alpha_2, ...\alpha_n$ and if P is a polynomial of degree $< n$, show that $\frac{P(x)}{Q(x)} = \sum_{k=1}^{n} \frac{P(\alpha_k)}{Q'(\alpha_k)(x - \alpha_k)}$. Hence prove that there exists a unique polynomial of degree with given values c_k at the point α_k, $k = 1, 2, ...n.$. **[30 Marks]**

99. Draw a flowchart and algorithm for solving the following system of 3 linear equations in 3 unknowns x_1, x_2 & x_3 : $C * X = D$ with $C = \left(c_{ij}\right)^3_{i,j=1}, X = \left(x_j\right)^3_{j=1}, D = \left(d_i\right)^3_{i=1}$ **[30 Marks]**

2005

100. Use appropriate quadrature formulae out of the Trapezoidal and Simpson's rules to numerically integrate $\int_0^1 \frac{dx}{1+x^2}$ with $h = 0.2$. Hence obtain an approximate value of π. Justify the use of particular quadrature formula. **[12 Marks]**

101. Find the hexadecimal equivalent of $(41819)_{10}$ and decimal equivalent of $(111011.10)_2$ **[12 Marks]**

102. Find the unique polynomial $P(x)$ of degree 2 or less such that $P(1) = 1, P(3) = 27, P(4) = 64$. Using the Lagrange's interpolation formula and the Newton's divided difference formula, evaluate $P(1.5)$
[30 Marks]

103. Draw a flow chart and also write algorithm to find one real root of the nonlinear equation $x = \phi(x)$ by the fixed-point iteration method. Illustrate it to find one real root, correct up to four places of decimals, of $x^3 - 2x - 5 = 0$. **[30 Marks]**

2004

104. The velocity of a particle at distance from a pint on it s path is given by the following table:

S(meters)	0	10	20	30	40	50	60

$V(\text{m/sec})$ 47 58 64 65 61 52 38

Estimate the time taken to travel the first 60 meters using Simpson's $\frac{1}{3}^{rd}$ rule. Compare the result with Simpson's $\frac{3}{8}^{th}$ rule. **[12 Marks]**

105. [i] If $(ABCD)_{16} = (x)_2 = (y)_8 = (z)_{10}$ then find $x, y \& z$

[ii] In a 4-bit representation, what is the value of 1111 in signed integer form, unsigned integer form, signed 1's complement form and signed 2's complement form? **[6+6=12 Marks]**

106. How many positive and negative roots of the equation $e^x - 5\sin x = 0$ exist? Find the smallest positive root correct to 3 decimals, using Newton-Raphson method. **[10 Marks]**

107. Using Gauss-Seidel iterative method, find the solution of the following system:

$$4x - y + 8z = 26$$
$$5x + 2y - z = 6$$
$$x - 10y + 2z = -13$$

up to three iterations. **[15 Marks]**

2003

108. Evaluate $\int_0^1 e^{-x^2} dx$ by employing three points Gaussian quadrature formula, finding the required weights and residues. Use five decimal places for computation. **[12 Marks]**

109. [i] Convert the following binary number into octal and hexa decimal system: 101110010.10010

[ii] Find the multiplication of the following binary numbers: 11001.1 and 101.1 **[6+6=12 Marks]**

110. Find the positive root of the equation $2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1}$ using Newton-Raphson method correct to four decimal places. Also show that the following scheme has error of second order:

$$x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2}\right)$$ **[30 Marks]**

111. Draw a flow chart and algorithm for Simpson's $\frac{1}{3}^{rd}$ rule for integration $\int_a^b \frac{1}{1+x^2} dx$ correct to 10^{-6} **[30 Marks]**

2002

112. Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by the method of false position. **[12 Marks]**

113. [i] Convert $(100.85)_{10}$ into its binary equivalent.

[ii] Multiply the binary numbers $(1111.01)_2$ and $(1101.11)_2$ and check with its decimal equivalent **[4+8=12 Marks]**

114. [i] Find the cubic polynomial which takes the following values: $y(0) = 1,\ y(1) = 0,\ y(2) = 1\ \&\ y(3) = 10$. Hence, or otherwise, obtain $y(4)$

[ii] Given: $\frac{dy}{dx} = y - x$ where $y(0) = 2$, using the Runge-Kutta fourth order method, find $y(0.1)$ and $y(0.2)$. Compare the approximate solution with its exact solution. $\left(e^{0.1} = 1.10517, e^{0.2} = 1.2214\right)$.

[10+20=30 Marks]

115. Draw a flow chart to examine whether a given number is a prime. **[10 Marks]**

2001

116. Show that the truncation error associated with linear interpolation of $f(x)$, using ordinates at x_0 and x_1 with $x_0 \le x \le x_1$ is not larger in magnitude than $\frac{1}{8}M_2(x_1 - x_0)^2$ where $M_2 = \max|f''(x)|$ in $x_0 \le x \le x_1$. Hence show that if $f(x) = \frac{2}{\sqrt{\pi}}\int_0^{\pi} e^{-t^2}dt,$ the truncation error corresponding to linear interpolation of $f(x)$ in $x_0 \le x \le x_1$ cannot exceed $\frac{(x_1 - x_0)^2}{2\sqrt{2\pi e}}$. **[12 Marks]**

117. [i] Given $A.B' + A'.B = C$ show that $A.C' + A'.C = B$

[ii] Express the area of the triangle having sides of lengths $6\sqrt{2},\ 12,\ 6\sqrt{2}$ units in binary number system. **[6+6=12 Marks]**

118. Using Gauss Seidel iterative method and the starting solution $x_1 = x_2 = x_3 = 0,$ determine the solution of the following system of equations in two iterations

$$10x_1 - x_2 - x_3 = 8$$
$$x_1 + 10x_2 + x_3 = 12.$$
$$x_1 - x_2 + 10x_3 = 10$$

Compare the approximate solution with the exact solution **[30 Marks]**

119. Find the values of the two-valued variables $A, B, C \,\&\, D$ by solving the set of simultaneous equations

$$A' + A.B = 0$$
$$A.B = A.C$$
$$A.B + A.C' + C.D = C'.D$$

[15 Marks]

2000

120. [i] Using Newton-Raphson method, show that the iteration formula for finding the reciprocal of the p^{th} root of N is $x_{i+1} = \frac{x_i\left(p + 1 - Nx_i\right)}{p}$

[ii] Prove De Morgan's Theorem $(p+q)' = p'.q'$ **[6+6=12 Marks]**

121. [i] Evaluate $\int_0^1 \frac{dx}{1+x^2}$, by subdividing the interval $(0,\ 1)$ into 6 equal parts and using Simpson's one-third rule. Hence find the value of π and actual error, correct to five places of decimals

[ii] Solve the following system of linear equations, using Gauss-elimination method:

$$x_1 + 6x_2 + 3x_3 = 6$$
$$2x_1 + 3x_2 + 3x_3 = 117$$
$$4x_1 + x_2 + 2x_3 = 283$$

[15+15=30 Marks]

1999

122. Obtain the Simpson's rule for the integral $I=\int_a^b f(x)dx$ and show that this rule is exact for polynomials of degree $n \le 3$. In general show that the error of approximation for Simpson's rule is given by

$$R=-\frac{(b-a)^5}{2880}f^{iv}(\eta),\ \eta\in(0,2).$$

Apply this rule to the integral $\int_0^1 \frac{dx}{1+x}$ and show that $|R| \le 0.008333.$ **[20 Marks]**

123. Using fourth order classical Runge-Kutta method for the initial value problem $\frac{du}{dt}=-2tu^2, u(0)=1,$ with $h=0.2$ on the interval $[0,\ 1]$, calculate $u(0.4)$ correct to six places of decimal. **[20 Marks]**

1998

124. Evaluate $\int_1^3 \frac{dx}{x}$ by Simpson's rule with 4 strips. Determine the error by direct integration. **[20 Marks]**

125. By the fourth –order Runge-Kutta method. tabulate the solution of the differential equation $\frac{dy}{dx}=\frac{xy+1}{10y^2+4}$, $y(0)=0$ in $[0,\ 0.4]$ with step length 0.1 correct to five places of decimals **[20 Marks]**

126. Use Regula-Falsi method to show that the real root of $x\log_{10}x-1.2=0$ lies between 3 and 2.740646 **[20 Marks]**

1997

127. Apply that fourth order Runge-Kutta method to find a value of y correct to four places of decimals at $x=0.2$, when $y'=\frac{dy}{dx}=x+y,\ y(0)=1$ **[20 Marks]**

128. Show that the iteration formula for finding the reciprocal of N is $x_{n+1}=x_n\left(2-N_{xn}\right),\ n=0,\ 1...$ **[20 Marks]**

129. Obtain the cubic spline approximation for the function given in the tabular form below:

x	0	1	2	3
$f(x)$	1	2	33	244

and $M_0=0, M_3=0$ **[20 Marks]**

1996

130. Describe Newton-Raphson method for finding the solutions of the equation $f(x)=0$ and show that the method has a quadratic convergence. **[20 Marks]**

131. The following are the measurements t made on a curve recorded by the oscillograph representing a change of current i due to a change in the conditions of an electric current:

t	1.2	2.0	2.5	3.0
i	1.36	0.58	0.34	0.20

Applying an appropriate formula interpolate for the value of i when $t=1.6$ **[20 Marks]**

132. Solve the system of differential equations $\frac{dy}{dx} = xz + 1, \; \frac{dz}{dx} = -xy$ for $x = 0.3$ given that $y = 0$ and $z = 1$ when $x = 0,$ using Runge-Kutta method of order four **[20 Marks]**

1995

133. Find the positive root of $\log_e x = \cos x$ nearest to five places of decimal by Newton-Raphson method. **[20 Marks]**

134. Find the value of $\int_{1.6}^{3.4} f(x)\, dx$ from the following data using Simpson's $\frac{3}{8}$ rd rule for the interval $(1.6,\ 2.2)$ and $\frac{1}{8}$ th rule for $(2.2,\ 3.4)$:

x	1.6	1.8	2.0	2.2	2.4
$f(x)$	4.953	6.050	7.389	9.025	11.023

x	2.6	2.8	3.0	3.2	3.4
$f(x)$	13.464	16.445	20.086	24.533	29.964

[20 Marks]

1994

135. Find the positive root of the equation $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} e^{0.3x}$ correct to five decimal places. **[20 Marks]**

136. Fit the following four points by the cubic splines.

i	0	1	2	3
x_i	1	2	3	4
y_i	1	5	11	8

Use the end conditions Use the end conditions $y''_0 = y''_3 = 0$

Hence compute [i] $y(1.5)$

[ii] $y'(2)$ **[20 Marks]**

137. Find the derivative of $f(x)$ at $x = 0.4$ from the following table:

x	0.1	0.2	0.3	0.4
$y = f(x)$	1.10517	1.22140	1.34986	1.49182

[20 Marks]

1993

138. Find correct to 3 decimal places the two positive roots of $2e^x - 3x^2 = 2.5644$ **[20 Marks]**

139. Evaluate approximately $\int_{-3}^{3} x^4 \, dx$ Simpson's rule by taking seven equidistant ordinates. Compare it with the value obtained by using the trapezoidal rule and with exact value. **[20 Marks]**

140. Solve $\frac{dy}{dx} = xy$ for $x = 1.4$ by Runge-Kutta method, initially $x = 1, y = 2$ (Take $h = 0.2$) **[20 Marks]**

1992

141. Compute to 4 decimal placed by using Newton-Raphson method, the real root of $x^2 + 4\sin x = 0$. **[20 Marks]**

142. Solve by Runge-Kutta method $\frac{dy}{dx} = x + y$ with the initial conditions $x_0 = 0, y_0 = 1$ correct up to 4 decimal places, by evaluating up to second increment of y (Take $h = 0.1$) **[20 Marks]**

143. Fit the natural cubic spline for the data. **[20 Marks]**

x:	0	1	2	3	4
y:	0	0	1	0	0