Ordinary Differential Equations (ODE)

Previous year Questions

from 2025 to 1992

2025

1. Solve $\left(1-y^2+\frac{y^4}{x^2}\right)\left(\frac{dy}{dx}\right)^2-2\frac{y}{x}\frac{dy}{dx}+\frac{y^2}{x^2}=0.$ **[10 Marks]**
2. Form the differential equation of all ellipses whose axes coincide with coordinate axes. **[10 Marks]**
3. If $F(s)$ and $G(s)$ are Laplace transforms of $f(t)$ and $g(t)$ respectively, then prove that $\mathcal{L}\left[\int_0^t f(x)g(t-x)\,dx\right]=F(s)G(s)$. Using this result, solve the equation $y(t)=t+\int_0^t y(x)\sin(t-x)\,dx$. **[15 Marks]**
4. Find the general solution and singular solution of the differential equation $\left(1+\frac{dy}{dx}\right)^3=\frac{27}{8a}(x+y)\left(1-\frac{dy}{dx}\right)^3$. **[10 Marks]**
5. Find the complete solution of $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x\log x$. **[10 Marks]**
6. Solve the differential equation $(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=(1+x)e^x$ by the method of variation of parameters. **[15 Marks]**

2024

7. Find the orthogonal trajectories of the family of curves $r=c(\sec\theta+\tan\theta)$, where c is a parameter. **[10 Marks]**
8. Solve the integral equation $y(t)=\cos t+\int_0^t y(x)\cos(t-x)\,dx$ using Laplace transform. **[10 Marks]**
9. Find the second solution of the differential equation $xy''+(x-1)y'-y=0$ using $u(x)=-e^{-x}$ as one of the solutions. **[10 Marks]**
10. Find the general solution of the differential equation $x^2y''-2xy'+2y=x^3\sin x$ by the method of variation of parameters. **[10 Marks]**
11. State uniqueness theorem for the existence of unique solution of the initial value problem $\frac{dy}{dx}=f(x,y),\ y(x_0)=y_0$ in the rectangular region $R:|x-x_0|\le a,\ |y-y_0|\le b$. Test the existence and uniqueness of the solution of $\frac{dy}{dx}=2\sqrt{y},\ y(1)=0$ in a suitable rectangle R. If more than one solution exist, then find all the solutions. **[15 Marks]**
12. Using Laplace transform, solve the initial value problem $y''+2y'+5y=\delta(t-2),\ y(0)=0,\ y'(0)=0$, where $\delta(t-2)$ denotes the Dirac delta function. **[15 Marks]**

2023

13. Obtain the solution of the initial-value problem $\frac{dy}{dx}-2xy=2,\ y(0)=1$ in the form $y=e^{x^2}[1+\sqrt{\pi}\,\mathrm{erf}(x)]$. **[10 Marks]**
14. Given that $\mathcal{L}\{f(t);p\}=F(p)$, show that $\int_0^\infty\frac{f(t)}{t}\,dt=\int_0^\infty F(x)\,dx$. Hence evaluate $\int_0^\infty\frac{e^{-t}-e^{-3t}}{t}\,dt$. **[10 Marks]**
15. Solve the differential equation $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$. **[15 Marks]**
16. Find the solution of the differential equation $\frac{dy}{dx}=-\frac{2xy^3+2}{3x^2y^2+8e^{4y}}$. **[10 Marks]**
17. Reduce the equation $x^2p^2+y(2x+y)p+y^2=0$ to Clairaut's form by the substitution $y=u$ and $xy=v$. Hence solve the equation and show that $y+4x=0$ is a singular solution of the differential equation. **[10 Marks]**
18. Solve the following initial value problem by using Laplace transform technique: $\frac{d^2y}{dt^2}-4\frac{dy}{dt}+3y(t)=f(t),\ y(0)=1,\ y'(0)=0$, where $f(t)$ is a given function of t. **[15 Marks]**

2022

19. Show that the general solution of the differential equation $\frac{dy}{dx} + Py = Q$ can be written in the form $y = \frac{Q}{P} - e^{-\int P\,dx}\left\{C + \int e^{\int P\,dx}\, d\left(\frac{Q}{P}\right)\right\}$, where P, Q are non-zero functions of x and C is an arbitrary constant. **[10 Marks]**

20. Show that the orthogonal trajectories of the system of parabolas $x^2 = 4a(y + a)$ belong to the same system. **[10 Marks]**

21. Solve the following differential equation by using the method of variation of parameters: $(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = (x^2 - 1)^2$, given that $y = x$ is one solution of the reduced equation. **[15 Marks]**

22. Solve the following initial value problem by using Laplace's transformation: $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = h(t)$, where $h(t) = \begin{cases} 2, & 0 < t < 4, \\ 0, & t > 4, \end{cases}$ $y(0) = 0,\ y'(0) = 0.$ **[15 Marks]**

23. Find the general and singular solutions of the differential equation $(x^2 - a^2)p^2 - 2xyp + y^2 + a^2 = 0$, where $p = \frac{dy}{dx}$. Also give the geometric relation between the general and singular solutions. **[10 Marks]**

24. Solve the following differential equation: $(3x + 2)^2\frac{d^2y}{dx^2} + 5(3x + 2)\frac{dy}{dx} - 3y = x^2 + x + 1.$ **[10 Marks]**

2021

25. Solve the differential equation $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x \cos 2x.$ **[10 Marks]**

26. Solve the initial value problem $\frac{d^2y}{dx^2} + 4y = e^{-2x}\sin 2x,\ y(0) = y'(0) = 0$ using Laplace transform method. **[10 Marks]**

27. Solve the equation $\frac{d^2y}{dx^2} + (\tan x - 3\cos x)\frac{dy}{dx} + 2y\cos^2 x = \cos^4 x$ completely by demonstrating all the steps involved. **[15 Marks]**

28. Find all possible solutions of the differential equation $y^2 \log y = xy\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2.$ **[15 Marks]**

29. Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, where $a > b > 0$ are constants and λ is a parameter. Show that the given family of curves is self-orthogonal. **[10 Marks]**

30. Find the general solution of the differential equation $x^2\frac{d^2y}{dx^2} - 2x(1 + x)\frac{dy}{dx} + 2(1 + x)y = 0$. Hence, solve $x^2\frac{d^2y}{dx^2} - 2x(1 + x)\frac{dy}{dx} + 2(1 + x)y = x^3$ by the method of variation of parameters. **[10 Marks]**

2020

31. Solve the following differential equation: $x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$ **[10 Marks]**

32. Find the orthogonal trajectories of the family of circles passing through the points $(0,2)$ and $(0,-2)$ **[10 Marks]**

33. Using the method of variation of parameters, solve the differential equation $y''+(1-\cot x)y'-y\cot x=\sin^2 x$ if $y=e^{-x}$ is one solution of CF. **[20 Marks]**

34. Using Laplace transform, solve the initial value problem $ty''+2ty'+2y=2;$ and $y(0)=1$ and $y'(0)$ is arbitrary. Does this problem have a unique solution? **[10 Marks]**

35. Solve the following differential equation: $(x+1)^2y''-4(x+1)y'+6y=6(x+1)^2+\sin\log(x+1)$ **[10 Marks]**

36. Find the general and singular solutions of the differential equation $9p^2(2+y)^2=4(3-y)$ where $p=\frac{dy}{dx}$ **[10 Marks]**

2019

37. Solve the differential equation $(2y\sin x+3y^4\sin x\cos x)dx-(4y^3\cos^2 x+\cos x)dy=0$ **[10 Marks]**

38. Determine the complete solution of the differential $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=3x^2e^{2x}\sin 2x$ **[10 Marks]**

39. Solve the differential equation $\frac{d^2y}{dx^2}+(3\sin x-\cot x)\frac{dy}{dx}+2y\sin^2 x=e^{-\cos x}\sin^2 x$ **[10 Marks]**

40. Find the Laplace transforms of $t^{-1/2}$ and $t^{-1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n\in N$ is $\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$ **[10 Marks]**

41. Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2y''-2xy'+2y=x^3\sin x$ and then find the general solution of the given equation by the method of variation of parameters. **[15 Marks]**

42. Obtain the singular solution of the differential equation $\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2\cot^2\alpha-2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\operatorname{cosec}^2\alpha=1$. Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solutions **[15 Marks]**

2018

43. Solve: $y''-y=x^2e^{2x}$ **[10 Marks]**

44. Solve: $y'''-6y''+12y'-8y=12e^{2x}+27e^{-x}$ **[10 Marks]**

45. (i) Find the Laplace transform of $f(t)=\frac{1}{\sqrt{t}}$. **[10 Marks]**

(ii) Find the Inverse Laplace transform of $\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}$

46. Solve: $\left(\frac{dy}{dx}\right)^2 y + 2\frac{dy}{dx}x - y = 0$ **[13 Marks]**

47. Solve: $y'' + 16y = 32\sec 2x$ **[13 Marks]**

48. Solve: $(1+x)^2 y'' + (1+x)y' + y = 4\cos(\log(1+x))$ **[13 Marks]**

49. Solve the initial value problem

$y'' - 5y' + 4y = e^{2t}; y(0) = \frac{19}{12}, y'(0) = \frac{8}{3}$ **[13 Marks]**

50. Find α and β such that $x^{\alpha} y^{\beta}$ an integrating factor of $(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ and solve the equation. **[12 Marks]**

51. Find $f(y)$ such that $(2xe^y + 3y^2)dy + (3x^2 + f(y))dx = 0$ is exact and hence solve. **[12 Marks]**

2017

52. Find the differential equation representing the entire circle in the xy-plane. **[10 Marks]**

53. Solve the following simultaneous liner differential equations: $(D+1)y = z + e^x$ and $(D+1)z = y + e^x$ where y and z are functions of independent variable x and $D \equiv \frac{d}{dx}$. **[8 Marks]**

54. If the growth rate of the population of bacteria at time t is proportional to the amount present at the time and population doubles in one week, then how much bacteria's can be expected after 4 weeks? **[8 Marks]**

55. Consider the differential equation $xyp^2 - (x^2 + y^2 - 1)p + xy = 0$ where $p = \frac{dy}{dx}$ substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p' = \frac{dv}{du}$ hence or otherwise solve the equation. **[10 Marks]**

56. Solve the following initial value differential equations $20y'' + 4y' + y = 0,\ y(0) = 3.2,\ y'(0) = 0.$ **[7 Marks]**

57. Solve the differential equation: $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3 y = 8x^3 \sin(x^2)$ **[9 Marks]**

58. Solve that following differential equation using method of variation of parameters

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 44 - 76x - 48x^2$. **[8 Marks]**

59. Solve the following initial value problem using Laplace transform: $\frac{d^2y}{dx^2} + 9y = r(x),\ y(0) = 0,\ y'(0) = 4$

where $r(x) = \begin{cases} 8\sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x \geq \pi \end{cases}$. **[17 Marks]**

2016

60. Find a particular integral of $\frac{d^2y}{dx^2} + y = e^{x/2}\sin\frac{x\sqrt{3}}{2}$ **[10 marks]**

61. Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. **[10 marks]**

62. Solve $\{y(1 - x\tan x) + x^2\cos x\}dx - xdy = 0$ **[10 marks]**

63. Using the method of variation of parameter solve the differential equation

$(D^2 + 2D + 1)y = e^{-x}\log(x),\quad \left[D \equiv \frac{d}{dx}\right]$ **[15 marks]**

64. Find the general solution of the equation $x^2 \frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$ **[15 marks]**

65. Using Laplace transformation solves the following: $y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6$ **[10 marks]**

2015

66. Solve the differential equation: $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$ **[10 Marks]**

67. Solve the differential equation: $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ **[10 Marks]**

68. Find the constant a so that $(x+y)^a$ is the integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential equation **[12 Marks]**

69. (i) Obtain Laplace Inverse transform of $\left\{\ln\left(1+\frac{1}{s^2}\right) + \frac{s}{s^2+25}e^{-5s}\right\}$

(ii) Using Laplace transform, solve $y'' + y = t, y(0) = 1, y'(0) = -2$ **[6+6=12 Marks]**

70. Solve the differential equation $x = py - p^2$ where $p = \frac{dy}{dx}$ **[13 Marks]**

71. Solve $x^4\frac{d^4y}{dx^4} + 6x^3\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\cos(\log_e x)$ **[13 Marks]**

2014

72. Justify that a differential equation of the form: $\left[y + xf(x^2+y^2)\right]dx + \left[yf(x^2+y^2) - x\right]dy = 0$ where $f(x^2+y^2)$ is an arbitrary function of (x^2+y^2), is not an exact differential equation and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2+y^2) = (x^2+y^2)^2$ **[10 Marks]**

73. Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency. **[10 Marks]**

74. Solve by the method of variation of parameters: $\frac{dy}{dx} - 5y = \sin x$ **(10 Marks]**

75. Solve the differential equation: $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log_e x)$ **[20 Marks]**

76. Solve the following differential equation: $x\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$, when e^x is a solution to its corresponding homogeneous differential equation. **[15 Marks]**

77. Find the sufficient condition for the differential equation $M(x,y)dx + N(x,y)dy = 0,$ to have an integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence find the integrating factor for the differential equation of $(x^2+xy)dx + (y^2+xy)dy = 0$ and solve it. **[15 Marks]**

78. Solve the initial value problem $\frac{d^2y}{dt^2} + y = 8e^{-2t}\sin t,\ y(0) = 0, y'(0) = 0$ by using Laplace transform. **[20 Marks]**

2013

79. If y is a function of x, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x+y)+\sin(x+y)$. Find out a relation between x and y, which is free from any derivative / differential. **[10 Marks]**

80. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a\sin n\theta,\ (r,\theta)$ being the plane polar coordinates. **[10 Marks]**

81. Solve the differential equation $(5x^3+12x^2+6y^2)dx+6xydy=0$ **[15 Marks]**

82. Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2}+a^2y=\sec ax$ **[15 Marks]**

83. Find the general solution of the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\ln x\sin(\ln x)$ **[15 Marks]**

84. By using Laplace transform method, solve the differential equation $(D^2+n^2)x=a\sin(nt+\alpha)$, $D^2=\frac{d^2}{dt^2}$ subject to the initial conditions $x=0$ and $\frac{dx}{dt}=0$, at $t=0$, in which a,n and α are constants. **[15 Marks]**

2012

85. Solve $\frac{dy}{dx}=\frac{2xye^{(x/y)^2}}{y^2(1+e^{(x/y)^2})+2x^2e^{(x/y)^2}}$ **[12 Marks]**

86. Find the orthogonal trajectory of the family of curves $x^2+y^2=ax$ **[12 Marks]**

87. Using Laplace transforms, solve the initial value problem $y''+2y'+y=e^{-t},\ y(0)=-1, y'(0)=1$ **[12 Marks]**

88. Show that the differential equation $(2xy\log y)dx+(x^2+y^2\sqrt{y^2+1})dy=0$ is not exact. Find an integrating factor and hence, the solution of the equation **[20 Marks]**

89. Find the general solution of the equation $y'''-y''=12x^2+6x$ **[20 Marks]**

90. Solve the ordinary differential equation $x(x-1)y''-(2x-1)y'+2y=x^2(2x-3)$ **[20 Marks]**

2011

91. Obtain the solution of the ordinary differential equation $\frac{dy}{dx}=(4x+y+1)^2$, if $y(0)=1$ **[10 Marks]**

92. Determine the orthogonal trajectory of a family of curves represented by the polar equation $r=a(1-\cos\theta)$, (r,θ) being the plane polar coordinates of any point. **[10 Marks]**

93. Obtain Clairaut's form of the differential equation $\left(x\frac{dy}{dx}-y\right)\left(y\frac{dy}{dx}+x\right)=a^2\frac{dy}{dx}$. Also find its general solution **[15 Marks]**

94. Obtain the general solution of the second order ordinary differential equation $y''-2y'+2y=x+e^x\cos x$, where dashes denote derivatives w.r.t. x **[15 Marks]**

95. Using the method of variation of parameters, solve the second order differential equation
$\frac{d^2y}{dx^2}+4y=\tan 2x$ **[15 Marks]**

96. Use Laplace transform method to solve the following initial value problem:
$\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t,\ x(0)=2$ and $\left.\frac{dy}{dt}\right|_{t=0}=-1$ **[15 Marks]**

2010

97. Consider the differential equation $y'=\alpha x,\ x>0$ where α is a constant. Show that
(i) If $\phi(x)$ is any solution and $\psi(x)=\phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;
(ii) If $\alpha<0,$ then every solution tends to zero as $x\to\infty$ **[12 Marks]**

98. Show that the differential equation $(3y^2-x)+2y(y^2-3)y'=0$ admits an integrating factor which is a function of $(x+y^2)$. Hence solve the equation **[12 Marks]**

99. Verify that $\frac{1}{2}(Mx+Ny)d\left[\log_e(xy)\right]+\frac{1}{2}(Mx-Ny)d\left[\log_e(x/y)\right]=Mdx+Ndy$. Hence show that-
(i) If the differential equation $Mdx+Ndy=0$ is homogeneous, then $(Mx+Ny)$ is an integrating factor unless $Mx+Ny\equiv 0$;
(ii) If the differential equation $Mdx+Ndy=0$ is not exact but is of the form $f_1(xy)ydx+f_2(xy)xdy=0$ then $(Mx-Ny)^{-1}$ is an integrating factor unless $Mx+Ny\equiv 0$;
[20 Marks]

100. Use the method of undermined coefficients to find the particular solutions of
$y''+y=\sin x+(1+x^2)e^x$ and hence find its general solution. **[20 Marks]**

2009

101. Find the Wronskian of the set of functions: $\left\{3x^3,\ \left|3x^3\right|\right\}$ on the interval $[-1,\ 1]$ and determine whether the set is linearly dependent on $[-1,\ 1]$ **[12 Marks]**

102. Find the differential equation of the family of circles in the xy-plane passing through $(-1,\ 1)$ and $(1,\ 1)$ **[20 Marks]**

103. Find the inverse Laplace transform of $F(s)=1n\left(\frac{s+1}{s+s}\right)$ **[20 Marks]**

104. Solve: $\frac{dy}{dx}=\frac{y^2(x-y)}{3xy^2-x^2y-4y^3},\ y(0)=1$ **[20 Marks]**

2008

105. Solve the differential equation $ydx+(x+x^3y^2)dy=0$ **[12 Marks]**

106. Use the method of variation of parameters to find the general solution of
$x^2y''-4xy'+6y=-x^4\sin x$ **[12 Marks]**

107. Using Laplace transform, solve the initial value problem $y''-3y'+2y=4t+e^{3t}$, $y(0)=1$, $y'(0)=-1$ **[15 Marks]**

108. Solve the differential equation $x^3y''-3x^2y'+xy=\sin(\ln x)+1$ **[15 Marks]**

109. Solve the equation $y-2xp+yp^2=0$, where $p=\frac{dy}{dx}$ **[15 Marks]**

2007

110. Solve the ordinary differential equation $\cos 3x\frac{dy}{dx}-3y\sin 3x=\frac{1}{2}\sin 6x+\sin^2 3x, 0<x<\frac{\pi}{2}$ **[12 Marks]**

111. Find the solution of the equation $\frac{dy}{y}+xy^2dx=-4xdx$ **[12 Marks]**

112. Determine the general and singular solutions of the equation $y=x\frac{dy}{dx}+a\frac{dy}{dx}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}$, a being a constant. **[15 Marks]**

113. Obtain the general solution of $[D^3-6D^2+12D-8]y=12\left(e^{2x}+\frac{9}{4}e^{-x}\right)$, where $D\equiv\frac{dy}{dx}$ **[15 Marks]**

114. Solve the equation $2x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-3y=x^3$ **[15 Marks]**

115. Use the method of variation of parameters to find the general solution of the equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=2e^x$ **[15 Marks]**

2006

116. Find the family of curves whose tangents form an angle $\frac{\pi}{4}$ with the hyperbolas $xy=c, c>0$ **[12 Marks]**

117. Solve the differential equation $\left(xy^2+e^{-\frac{1}{x^3}}\right)dx-x^2ydy=0$ **[12 Marks]**

118. Solve: $(1+y^2)+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0$ **[15 Marks]**

119. Solve the equation $x^2p^2+py(2x+y)+y^2=0$ using the substitution $y=u$ and $xy=v$ and find its singular solution, where $p=\frac{dy}{dx}$ **[15 Marks]**

120. Solve the differential equation $x^2\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}+2\frac{y}{x}=10\left(1+\frac{1}{x^2}\right)$ **[15 Marks]**

121. Solve the differential equation $(D^2-2D+2)y=e^x\tan x$, $D\equiv\frac{dy}{dx}$ by the method of variation of parameters. **[15 Marks]**

2005

122. Find the orthogonal trajectory of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter. **[12 Marks]**

123. Solve: $xy\dfrac{dy}{dx} = \sqrt{(x^2 - y^2 - x^2y^2 - 1)}$ **[12 Marks]**

124. Solve the differential equation: $\left[(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)\right] y = \dfrac{1}{(x+1)}$ **[15 Marks]**

125. Solve the differential equation: $(x^2 + y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$ where $p = \dfrac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. **[15 Marks]**

126. Solve the differential equation $(\sin x - x\cos x)y'' - x\sin x\ y' + y\sin x = 0$ given that $y = \sin x$ is a solution of this equation. **[15 Marks]**

127. Solve the differential equation $x^2y'' - 2xy' + 2y = x\log x,\ x > 0$ by variation of parameters **[15 Marks]**

2004

128. Find the solution of the following differential equation $\dfrac{dy}{dx} + y\cos x = \dfrac{1}{2}\sin 2x$ **[12 Marks]**

129. Solve: $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$ **[12 Marks]**

130. Solve: $(D^4 - 4D^2 - 5)y = e^x(x + \cos x)$ **[15 Marks]**

131. Reduce the equation $(px - y)(py + x) = 2p$, where $p = \dfrac{dy}{dx}$ to Clairaut's equation and hence solve it. **[15 Marks]**

132. Solve: $(x+2)\dfrac{d^2y}{dx^2} - (2x+5)\dfrac{dy}{dx} + 2y = (x+1)e^x$ **[15 Marks]**

133. Solve the following differential equation: $(1 - x^2)\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} - (1 + x^2)y = x$ **[15 Marks]**

2003

134. Show that the orthogonal trajectory of a system of con-focal ellipses is self-orthogonal **[12 Marks]**

135. Solve: $x\dfrac{dy}{dx} + y\log y = xye^x$ **[12 Marks]**

136. Solve $(D^5 - D) = 4(e^x + \cos x + x^3)$, where $D \equiv \dfrac{dy}{dx}$ **[15 Marks]**

137. Solve the differential equation $(px^2 + y^2)(px + y) = (P+1)^2$, where $p = \dfrac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions **[15 Marks]**

138. Solve $(1 - x^2)y'' + (1+x)y' + y = \sin 2\left[\log(1+x)\right]$ **[15 Marks]**

139. Solve the differential equation $x^2y'' - 4x\ y' + 6y = x^4\sec^2 x$ by variation of parameters. **[15 Marks]**

2002

140. Solve: $x\frac{dy}{dx}+3y=x^3y^2$ **[12 Marks]**

141. Find the values of λ for which all solutions of $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-\lambda y=0$ tend to zero as $x\to\infty$.
[12 Marks]

142. Find the value of constant λ such that the following differential equation becomes exact.
$(2xe^y+3y^2)\frac{dy}{dx}+(3x^2+\lambda e^y)=0$. Further, for this value of λ, solve the equation. **[15 Marks]**

143. Solve: $\frac{dy}{dx}=\frac{x+y+4}{x-y-6}$ **[15 Marks]**

144. Using the method of variation of parameters, find the solutions of $\frac{d^2y}{d^2x}-2\frac{dy}{dx}+y=xe^x\sin x$ with
$y(0)=0$ and $\left(\frac{dy}{dx}\right)_{x=0}=0$ **[15 Marks]**

145. Solve: $(D-1)(D^2-2D+2)y=e^x$ where $D\equiv\frac{dy}{dx}$ **[15 Marks]**

2001

146. A continuous function $y(t)$ satisfies the differential equation $\frac{dy}{dx}=\begin{cases}1+e^{1-t}, & 0\le t<1\\ 2+2t-3t^2, & 1\le t<5\end{cases}$ If
$y(0)=-e$ find $y(2)$ **[12 Marks]**

147. Solve: $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log_e x$ **[12 Marks]**

148. Solve: $\frac{dy}{dx}+\frac{y}{x}\log_e y=\frac{y(\log_e y)^2}{x^2}$ **[15 Marks]**

149. Find the general solution of $ayp^2+(2x-b)p-y=0,\ a>0$ **[15 Marks]**

150. Solve: $(D^2+1)^2y=24x\cos x$ given that $y=Dy=D^2y=0$ and $D^3y=12$ when $x=0$ **[15 Marks]**

151. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+4y=4\tan 2x$ **[15 Marks]**

2000

152. Show that $3\frac{d^2y}{dx^2}+4x\frac{dy}{dx}-8y=0$ has an integral which is a polynomial in x. Deduce the general solution. **[12 Marks]**

153. Reduce $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$, where P,Q,R are functions of x, to the normal form. Hence solve
$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$ **[15 Marks]**

154. Solve the differential equation $y = x - 2ap + ap^2$. Find the singular solution and interpret it geometrically **[15 Marks]**

155. Show that $(4x+3y+1)dx + (3x+2y+1)dy = 0$ represents a family of hyperbolas with a common axis and tangent at the vertex **[15 Marks]**

156. Solve $x\frac{dy}{dx} - y = (x-1)\left(\frac{d^2y}{dx^2} - x + 1\right)$ by the method of parameters **[15 Marks]**

1999

157. Solve the differential equation $\frac{xdx + ydy}{xdy - ydx} = \left(\frac{1-x^2-y^2}{x^2+y^2}\right)^{1/2}$ **[20 Marks]**

158. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$ **[20 Marks]**

159. By the method of variation of parameters solve the differential equation $\frac{d^2y}{dx^2} + a^2y = \sec(ax)$ **[20 Marks]**

1998

160. Solve the differential equation: $xy - \left(\frac{dy}{dx}\right) = y^3e^{-x^2}$ **[20 Marks]**

161. Show that the equation: $(4x+3y+1)dx + (3x+2y+1)dy = 0$ represents a family of hyperbolas having as asymptotes the lines $x + y = 0,\ 2x + y + 1 = 0$. **[20 Marks]**

162. Solve the differential equation: $y = 3px + 4p^2$ **[20 Marks]**

163. Solve the differential equation: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}(x^2+9)$ **[20 Marks]**

164. Solve the differential equation: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x\sin x$ **[20 Marks]**

165. Solve the differential equation: $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ **[20 Marks]**

1997

166. Solve the initial value problem $\frac{dy}{dx} = \frac{x}{x^2y + y^3}$, $y(0) = 0$ **[20 Marks]**

167. Solve $(x^2 - y^2 + 3x - y)dx + (x^2 - y^2 + x - 3y)dy = 0$ **[20 Marks]**

168. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. If its radius originally is 3mm, and one hour later has been reduced to 2 mm. find an expression for the radius of the rain drop at any time. **[20 Marks]**

169. Solve $\frac{d^4y}{dx^4} + 6\frac{d^3y}{dx^3} + 11\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 20e^{-2x}\sin x$ **[20 Marks]**

170. Make use of the transformation $y(x)=u(x)\sec x$ to obtain the solution of $y''-2y'\tan x+5y=0$, $y(0)=0,\ y'(0)=\sqrt{6}$ **[20 Marks]**

171. Solve $(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2,\ y(0)=0,\ y'(0)=2$ **[20 Marks]**

1996

172. Find the curves for which the sum of the reciprocals of the radius vector and polar sub tangent is constant. **[20 Marks]**

173. Solve: $x^2(y-px)=yp^2,\ p\equiv\frac{dy}{dx}$ **[20 Marks]**

174. Solve: $y\sin 2x dx-\left(1+y^2+\cos^2 x\right)dy=0$ **[20 Marks]**

175. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y+37\sin 3x=0$. Find the value of y when $x=\frac{\pi}{2}$, if it is given that $y=3$ and $\frac{dy}{dx}=0$ when $x=0$ **[20 Marks]**

176. Solve: $\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}=x^2+3e^{2x}+4\sin x$ **[20 Marks]**

177. Solve: $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x+\log x$ **[20 Marks]**

1995

178. Determine a family of curves for which the ratio of the y- intercept of the tangent to the radius vector is a constant. **[20 Marks]**

179. Solve $(2x^2+3y^2-7)xdx+(3x^2+2y^2-8)ydy=0$ **[20 Marks]**

180. Test whether the equation $(x+y)^2dx-(y^2-2xy-x^2)dy=0$ is exact and hence solve it. **[20 Marks]**

181. Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$ **[20 Marks]**

182. Determine all real valued solutions of the equations $y'''-iy''+y'-iy=0,\ y'=\frac{dy}{dx}$ **[20 Marks]**

183. Find the solution of the equation $\frac{d^2y}{dx^2}+4y=8\cos 2x,$ given that $y=0$ and $y'=2$ when $x=0$ **[20 Marks]**

1994

184. Solve: $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ **[20 Marks]**

185. Show that if $\frac{1}{Q}\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)$ is a function of x only, say, $f(x)$, then $F(x)=e^{\int f(x)dx}$ is an integration factor of $Pdx+Qdy=0$ **[20 Marks]**

186. Find the family of curves whose tangent from an angel $\frac{\pi}{4}$ with the hyperbola $xy = c$ **[20 Marks]**

187. Transform the differential equation $\frac{d^2y}{dx^2}\cos x + \frac{dy}{dx}\sin x - 2y\cos^3 x = 2\cos^5 x$ into one having z as independent variable where $z = \sin x$ and solve it. **[20 Marks]**

188. If $\frac{d^2x}{dt^2} + \frac{g}{b}(x-a) = 0$ (a, b and g being positive constants) and $x = a'$ and $\frac{dx}{dt} = 0$ when $t = 0$, show that $x = a + (a' - a)\cos t\sqrt{\frac{g}{b}t}$ **[20 Marks]**

189. Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}\sin 2x$ where $D \equiv \frac{dy}{dx}$ **[20 Marks]**

1993

190. Determine the curvature for which the radius of curvature is proportional to the slope of the tangent. **[20 Marks]**

191. Show that the system of co focal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self-orthogonal. **[20 Marks]**

192. Solve $\left\{y\left(1+\frac{1}{x}\right) + \cos y\right\}dx + \left(x + \log x - x\sin y\right)dy = 0$ **[20 Marks]**

193. Solve $y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 = y^2$ **[20 Marks]**

194. Solve $\frac{d^2y}{dx^2} + \omega_0{}^2 y = a\cos\omega t$ and discuss the nature of solution as $\omega \xrightarrow{dt^2} \omega_0$ **[20 Marks]**

195. Solve $(D^4 + D^2 + 1)y = e^{-x/2}\cos\left(x\frac{\sqrt{3}}{2}\right)$ **[20 Marks]**

1992

196. By eliminating the constants a, b obtain the differential equation for which $xy = ae^x + be^{-x} + x^2$ is a solution. **[20 Marks]**

197. Find the orthogonal trajectory of the family of semi cubical parabolas $ay^2 = x^3$, where a is a variable parameter. **[20 Marks]**

198. Show that $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents hyperbolas having the following lines as asymptotes $x + y = 0,\ 2x + y + 1 = 0$ **[20 Marks]**

199. Solve the following differential equation $y(1 + xy)dx + x(1 - xy)dy = 0$ **[20 Marks]**

200. Find the curves for which the portion of y- axis cut off between the origin and the tangent varies as the cube of the abscissa of the point of contact. **[20 Marks]**

201. Solve the following differential equation: $(D^2 + 4)y = \sin 2x$, given that when $x = 0$, then $y = 0$ and $\frac{dy}{dx} = 2$ **[20 Marks]**

202. Solve: $(D^3 - 1)y = xe^x + \cos^2 x$ **[20 Marks]**

203. Solve: $(x^2D^2 + xD - 4)y = x^2$ **[20 Marks]**