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Partial Differential Equations (PDE)

IFoS (IFS) Previous Year Questions (PYQ) from 2025 to 2009

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IAS, UPSC, IFS, IFoS, CIVIL SERVICE MAINS EXAMS MATHS OPTIONAL STUDY MATERIALS

2025

1. Eliminate the arbitrary function F from the given equation **[8 Marks]**

$$F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$$

and find the corresponding partial differential equation.

2. Find a complete integral of the equation **[10 Marks]**

$$p^2x + q^2y = z, \qquad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}.$$

3. Find the general solution of the partial differential equation **[15 Marks]**

$$\left(4D^2 - 4DD' + D'^2\right)z = 8\log(x+2y) + \cos 2x \cos y,$$

where $D = \dfrac{\partial}{\partial x}$ and $D' = \dfrac{\partial}{\partial y}$.

4. Let $u(x,t)$ be the solution of the wave equation **[15 Marks]**

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \qquad 0 < x < \pi, \quad t > 0,$$

with the initial conditions $u(x,0) = \sin x + \sin 2x + \sin 3x$, $\dfrac{\partial u}{\partial t}(x,0) = 0$, $0 < x < \pi$, and the boundary condition $u(0,t) = u(\pi,t) = 0$, $t \geq 0$. Find the value of $u\left(\dfrac{\pi}{2}, \pi\right)$.

2024

5. Obtain the partial differential equation by eliminating the arbitrary function f from the equation **[8 Marks]**

$$f\left(x+y+z,\ x^2+y^2+z^2\right) = 0.$$

6. Using Charpit's method, find the complete integral of **[10 Marks]**

$$yq + 3xp = 2\left(z - y^2p^2\right), \qquad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}.$$

7. Find the general solution of the partial differential equation **[15 Marks]**

$$\left[D^2 - (D')^2 - 3D + 3D'\right]z = (1-x)(1-y) + e^x + 2y,$$

where $D = \dfrac{\partial}{\partial x}$ and $D' = \dfrac{\partial}{\partial y}$.

8. Find the solution of the heat equation **[15 Marks]**

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < \pi, \quad t > 0,$$

under the boundary conditions $u(0,t) = 0 = u(\pi,t)$ and the initial condition

$$u(x,0) = \begin{cases} x, & 0 \le x < \dfrac{\pi}{2}, \\ \pi - x, & \dfrac{\pi}{2} \le x \le \pi. \end{cases}$$

2023

9. Solve **[8 Marks]**

$$u_{xx} + \frac{10}{3}u_{xy} + u_{yy} = -\sin(x+y).$$

10. Find the solution of **[15 Marks]**

$$u_x - uu_y + u = 0$$

for the initial values $x_0(s) = 0$, $y_0(s) = s$, $u_0(s) = -2s$. Does the solution break down for any finite x? Is the solution unique?

11. Solve **[10 Marks]**

$$u_{tt} - u_{xx} = 0, \qquad 0 < x < 2, \quad t > 0,$$

$$u(0,t) = u(2,t) = 0, \qquad u(x,0) = \sin^3\left(\frac{\pi x}{2}\right), \qquad u_t(x,0) = 0.$$

12. Show that **[15 Marks]**

$$f(x,y,z,p,q) = x^2p^2 + y^2q^2 - 4 = 0$$

and

$$g(x,y,z,p,q) = qy - a = 0,$$

where a is a constant, are compatible and hence solve $f(x,y,z,p,q) = 0$. Is it a complete integral?

2022

13. Equation of any cone with vertex at the point (a,b,c) is of the form **[8 Marks]**

$$f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0.$$

Find the partial differential equation of the cone.

14. Verify that the equation **[8 Marks]**

$$yz(y+z)\,dx + xz(x+z)\,dy + xy(x+y)\,dz = 0$$

is integrable and find its solution.

15. Find the system of equations for obtaining the general equation of surfaces orthogonal to the family given by **[10 Marks]**

$$x(x^2 + y^2 + z^2) = Cy^2,$$

where C is a parameter.

16. Find the solution of the partial differential equation **[10 Marks]**

$$z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y); \qquad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y},$$

which passes through the x-axis, using Cauchy's method of characteristics.

17. Find a complete integral of the partial differential equation **[15 Marks]**

$$(p^2 + q^2)x = pz; \qquad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y},$$

using Charpit's method and hence deduce the solution which passes through the curve $x = 0$, $z^2 = 4y$.

2021

18. Find the complete primitive of **[8 Marks]**

$$4r - 4s + t = 16 \log_e(x + 2y),$$

r, s, t bear their usual meanings.

19. Find the orthogonal trajectory of the following family of curves: **[8 Marks]**

$$x^2 - y^2 = a^2.$$

Then sketch the two families to demonstrate whether they cut orthogonally.

20. Solve the following by Charpit's method: **[10 Marks]**

$$pxy + pq + qy = yz, \qquad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}.$$

21. Solve the following differential equation: **[10 Marks]**

$$(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0, \qquad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}.$$

22. Find the singular solution of **[15 Marks]**

$$yp^2 - 2xp + y = 0.$$

Also trace the graph.