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Partial Differential Equations (P.D.E)

Previous year Questions from 2025 to 1992

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2025

1. Find the solution of the equation **[10 Marks]**

$$(D^2 + DD' - 2D'^2)z = y\sin x, \qquad D = \frac{\partial}{\partial x}, \qquad D' = \frac{\partial}{\partial y}.$$

2. Solve for a rectangular plate, subject to the boundary conditions **[20 Marks]**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

$$u(0, y) = 0, \qquad u(a, y) = 0, \qquad u(x, 0) = 0, \qquad u(x, b) = f(x).$$

3. Find the complete integral of **[15 Marks]**

$$z(p^2 - q^2) = x - y, \qquad p = \frac{\partial z}{\partial x}, \qquad q = \frac{\partial z}{\partial y}.$$

4. Find the characteristics of the partial differential equation **[15 Marks]**

$$p^2 + q^2 = 2, \qquad p = \frac{\partial z}{\partial x}, \qquad q = \frac{\partial z}{\partial y},$$

and determine the integral surface which passes through $x = 0,\ z = y$.

2024

5. Show that if f and g are arbitrary functions of their respective arguments, then **[10 Marks]**

$$u = f(x - kt + i\alpha y) + g(x - kt - i\alpha y)$$

is a solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, \qquad \alpha^2 = 1 - \frac{k^2}{c^2}.$$

6. Show that the solution of the two-dimensional Laplace equation **[20 Marks]**

$$\frac{\partial^2 \phi(x, y)}{\partial x^2} + \frac{\partial^2 \phi(x, y)}{\partial y^2} = 0, \qquad x \in (-\infty, \infty),\ y \geq 0,$$

subject to the boundary condition $\phi(x, 0) = f(x),\ x \in (-\infty, \infty)$, along with $\phi(x, y) \to 0$ for $|x| \to \infty$ and $y \to \infty$, can be written in the form

$$\phi(x, y) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{f(\xi)\, d\xi}{y^2 + (x - \xi)^2}.$$

7. Find the integral surface of the following quasi-linear equation **[15 Marks]**

$$(y - z)\frac{\partial z}{\partial x} + (z - x)\frac{\partial z}{\partial y} = x - y,$$

which passes through the curve $z = 0,\ xy = 1$ and through the circle

$$x + y + z = 0, \qquad x^2 + y^2 + z^2 = a^2.$$

8. Solve the partial differential equation by transforming it to the canonical form: **[15 Marks]**

$$\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial x} + \phi\right) + 2x^2 y\left(\frac{\partial \phi}{\partial x} + \phi\right) = 0.$$

2023

9. By eliminating the arbitrary functions f and g from **[10 Marks]**

$$z = f(x^2 - y) + g(x^2 + y),$$

form the partial differential equation.

10. Find the surface passing through the two lines $z = x = 0$ and $z - 1 = x - y = 0$, **[15 Marks]** and satisfying the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0.$$

11. Solve the partial differential equation **[20 Marks]**

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \qquad 0 < x < L,\ t > 0,$$

subject to the conditions

$$u(0,t) = 0, \qquad u(L,t) = 0, \qquad t > 0,$$

$$u(x,0) = x, \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 1, \qquad 0 < x < L.$$

12. Reduce the partial differential equation to canonical form: **[15 Marks]**

$$\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} - \left(1 + \frac{1}{x}\right)\frac{\partial z}{\partial y} + \frac{z}{x} = 0.$$

2022

13. It is given that the equation of any cone with vertex at (a, b, c) is **[10 Marks]**

$$f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0.$$

Find the differential equation of the cone.

14. Solve the heat equation **[20 Marks]**

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < l,\ t > 0,$$

subject to the conditions

$$u(0,t) = u(l,t) = 0, \qquad u(x,0) = x(l-x), \qquad 0 < x < l.$$

15. Find the general solution of the partial differential equation **[15 Marks]**

$$(D^2 + DD' - 6D'^2)z = x^2 \sin(x+y), \qquad D = \frac{\partial}{\partial x}, \qquad D' = \frac{\partial}{\partial y}.$$

16. Reduce the following partial differential equation to a canonical form and hence **[15 Marks]** solve it:

$$yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0.$$

2021

17. Obtain the partial differential equation by eliminating the arbitrary function f from the equation **[10 Marks]**

$$f(x+y+z,\ x^2+y^2+z^2)=0.$$

18. Solve the wave equation **[20 Marks]**

$$a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}, \qquad 0<x<L,\ t>0,$$

subject to the conditions

$$u(0,t)=0, \qquad u(L,t)=0,$$

$$u(x,0)=\frac{1}{4}x(L-x), \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0}=0.$$

19. Find the general solution of the partial differential equation **[15 Marks]**

$$(D^2-D'^2-3D+3D')z=xy+e^{x+2y}, \qquad D=\frac{\partial}{\partial x}, \qquad D'=\frac{\partial}{\partial y}.$$

20. Find a complete integral of the partial differential equation **[15 Marks]**

$$p=(z+qy)^2$$

by using Charpit's method.

2020

21. Form a partial differential equation by eliminating the arbitrary functions $f(x)$ and $g(y)$ from $z=yf(x)+xg(y)$ and specify its nature (elliptic, hyperbolic or parabolic) in the region $x\geq 0, y\geq 0$ **[10 Marks]**

22. Solve the partial differential equation: $(D^3-2D^2D'-DD'^2+2D'^3)z=e^{2x+y}+\sin(x-2y)$; $D=\frac{\partial}{\partial x}, D'=\frac{\partial}{\partial y}$ **[10 Marks]**

23. Find the integral surface of the partial differential equation: $(x-y)y^2\frac{\partial z}{\partial x}+(y-x)x^2\frac{\partial z}{\partial y}=(x^2+y^2)z$ that contains the curve : $xz=a^3, y=0$ on it. **[15 Marks]**

24. Find the solution of the partial differential equation: $z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y);\ p=\frac{\partial z}{\partial x},\ q\frac{\partial z}{\partial y}$ which passes through the x-axis. **[15 Marks]**

2019

25. Form a partial differential equation of the family of surface given by the following expression. $\psi(x^2+y^2+2z^2, y^2-2zx)=0$ **[10 Marks]**

26. Solve the first order quasi linear partial differential equation by the method of characteristics: $x\frac{\partial u}{\partial x}+(u-x-y)\frac{\partial u}{\partial y}=x+2y$ in $x\geq 0, -\infty<y<\infty$ with $u=1+y$ on $x=1$ **[15 Marks]**

27. Reduce the following second order partial differential equations to canonical form and find the general solution: $\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y} = 12x$ **[20 Marks]**

2018

28. Find the partial differential equation of the family of all tangent planes to the ellipsoid $x^2 + 4y^2 + 4z^2 = 4$ which are not perpendicular to the xy plane. **[10 Marks]**

29. Find the general solution of the partial differential equation: $(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$, and find its integral surface that passes through the curve: $x = t, y = t^2, z = 1.$

[15 Marks]

30. Solve the partial differential equation: $(2D^2 - 5DD' + 2D'^2)Z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$ Where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}.$ **[15 Marks]**

2017

31. Solve $(D^2 - 2DD' - D'^2)z = e^{x+2y} + x^3 + \sin 2x$ where $D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y},\ D^2 \equiv \frac{\partial^2}{\partial x^2},\ D'^2 \equiv \frac{\partial^2}{\partial y^2}.$ **[10 Marks]**

32. Let Γ be a closed curve in xy- plane and let S denote the region bounded by the curve $\Gamma.$ Let $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x, y)\ \ \forall (x, y) \in S.$ If f is prescribed at each point (x, y) of S and w is prescribed on the boundary Γ of S then prove that any solution $w = w(x, y),$ satisfying these conditions, is unique. **[10 Marks]**

33. Find a complete integral of the partial differential equation $2(pq + yp + qx) + x^2 + y^2 = 0.$ **[15 Marks]**

34. Reduce the equation $y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and hence solve it.

[15 Marks]

35. Given the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2};\ t > 0,$ where $c^2 = \frac{T}{m},\ T$ the constant tension in the string and m is the mass per unit length of the string.
    (i) Find the appropriate solution of the wave equation
    (ii) Find also the solution under the conditions $y(0,t) = 0,\ y(l,t) = 0$ for all t and $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0,\ y(x,0) = a\sin\frac{\pi x}{t}, 0 < x < l,\ a > 0$. **[20 Marks]**

2016

36. Find the general equation of surfaces orthogonal to the family of spheres given by $x^2 + y^2 + z^2 = cz.$
[10 Marks]

37. Final he general integral of the particle differential equation $(y + zx)p - (x + yz)q = x^2 - y^2$
[10 Marks]

38. Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes though the parabola $4z + x^2 = 0$ **[15 Marks]**

39. Solve the particle differential equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y}\frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$ **[15 Marks]**

40. Find the temperature $u(x,t)$ in a bar of silver of length and constant cross section of area $1cm^2$. Let density $p = 10.6g / cm^3$, thermal conductivity $K = 1.04 / (cm \sec^\circ C)$ and specific heat $\sigma = 0.056 / g^\circ C$ the bar is perfectly isolated laterally with ends kept at $0^\circ C$ and initial temperature $f(x) = \sin(0.1\pi x)^\circ C$ note that $u(x,t)$ follows the head equation $u_t = c^2 u_{xx}$ where $c^2 = k / (\rho\sigma)$ **[20 Marks]**

2015

41. Solve the partial differential equation: $(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ **[10 Marks]**

42. Solve: $(D^2 + DD' - 2D'^2)u = e^{x+y}$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$ **[10 Marks]**

43. Solve for the general solution $p\cos(x+y) + q\sin(x+y) = z$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ **[15 Marks]**

44. Find the solution of the initial-boundary value problem

$$u_t - u_{xx} + u = 0, \quad 0 < x < l,\ t > 0$$
$$u(0,\ t) = u(l,\ t) = 0, \quad t \ge 0$$
$$u(x,\ 0) = x(l - x), \quad 0 < x < l$$

[15 Marks]

45. Reduce the second-order partial differential equation $x^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ into canonical form. Hence, find its general solution **[15 Marks]**

2014

46. Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$ **[10 Marks]**

47. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2\frac{\partial^2 z}{\partial y^2}$ to canonical form. **[15 Marks]**

48. Find the deflection of a vibrating string (length= π, ends fixed, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$) corresponding to zero initial velocity and initial deflection. $f(x) = k(\sin x - \sin 2x)$ **[15 Marks]**

49. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1,\ t > 0$, given that

(i) $u(x,\ 0) = 0, \quad 0 \le x \le 1;$

(ii) $\frac{\partial u}{\partial t}(x,\ 0) = x^2, \quad 0 \le x \le 1$

(iii) $u(0,\ t) = u(1,\ t) = 0,$ for all t **[15 Marks]**

2013

50. From a partial differential equation by eliminating the arbitrary functions f and g from $z = yf(x) + xg(y)$ **[10 Marks]**

51. Reduce the equation $y\dfrac{\partial^2 z}{\partial x^2} + (x+y)\dfrac{\partial^2 z}{\partial x \partial y} + x\dfrac{\partial^2 z}{\partial y^2} = 0$ to its canonical from when $x \neq y$ **[10 Marks]**

52. Solve $(D^2 + DD' - 6D'^2)z = x^2 \sin(x+y)$ where D and D' denote $\dfrac{\partial}{\partial x}$ and $\dfrac{\partial}{\partial y}$ **[15 Marks]**

53. Find the surface which intersects the surfaces of the system $z(x+y) = C(3z+1)$, (C being a constant) orthogonally and which passes through the circle $x^2 + y^2 = 1,\ z = 1$ **[15 Marks]**

54. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity $\lambda . x(l - x)$, find the displacement of the string at any distance x from one end at any time t **[20 Marks]**

2012

55. Solve partial differential equation $(D - 2D')(D - D')^2 z = e^{x+y}$ **[12 Marks]**

56. Solve partial differential equation $px + qy = 3z$ **[20 Marks]**

57. A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then released from rest. Find the deflection $y(x,\ t)$ of the vibrating string. **[20 Marks]**

58. The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. **[20 Marks]**

2011

59. Solve the PDE $(D^2 - D'^2 + D + 3D' - 2)z = e^{(x-y)} - x^2 y$ **[12 Marks]**

60. Solve the PDE $(x+2z)\dfrac{\partial z}{\partial x} + (4zx - y)\dfrac{\partial z}{\partial y} = 2x^2 + y$ **[12 Marks]**

61. Find the surface satisfying $\dfrac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$ **[20 Marks]**

62. Solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0,\ 0 \le x \le a,\ 0 \le y \le b$ satisfying the boundary conditions

$$u(0,\ y) = 0,\ u(x,\ 0) = 0,\ u(x,\ b) = 0\ \frac{\partial u}{\partial x}(a,\ y) = T\sin^3 \frac{\pi y}{a}$$ **[20 Marks]**

63. Obtain temperature distribution $y(x,\ t)$ in a uniform bar of unit length whose one end is kept at 10^0 and the other end is insulated. Also it is given that $y(x,\ 0) = 1 - x,\ 0 < x < 1$ **[20 Marks]**

2010

64. Solve the PDE $(D^2 - D')(D-2D')Z = e^{2x+y} + xy$ **[12 Marks]**

65. Find the surface satisfying the PDE $(D^2 - 2DD' + D'^2)Z = 0$ and the conditions that $bZ = y^2$ when $x = 0$ and $aZ = x^2$ when $y = 0$ **[12 Marks]**

66. Solve the following partial differential equation

$$zp + yq = x$$
$$x_0(s) = s,\ y_0(s) = 1,\ z_0(s) = 2s$$

by the method of characteristics. **[20 Marks]**

67. Reduce the following 2nd order partial differential equation into canonical form and find its general solution. $xu_{xx} + 2x^2 u_{xy} - u_x = 0$ **[20 Marks]**

68. Solve the following heat equation

$$u_t - u_{xx} = 0, \qquad 0 < x < 2,\ t > 0$$
$$u(0,\ t) = u(2,\ t) = 0 \quad t > 0$$
$$u(x,\ 0) = x(2-x), \quad 0 \le x \le 2$$

[20 Marks]

2009

69. Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that this equation is satisfied by the surface $yz + zx + xy = 0$. **[12 Marks]**

70. (i) Form the partial differential equation by elimination the arbitrary function f given by: $f(x^2 + y^2, z - xy) = 0$ **[20 Marks]**

(ii) Find the integral surface of: $x^2 p + y^2 q + z^2 = 0$ which passes through the curve: $xy = x + y,\ z = 1$ **[20 Marks]**

71. Find the characteristics of: $y^2 r - x^2 t = 0$ where r and t have their usual meanings. **[15 Marks]**

72. Solve: $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$ where D and D' represent $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ **[15 Marks]**

73. A tightly stretched string has its ends fixed at $x = 0$ and $x = 1$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at time $t > 0$. **[30 Marks]**

2008

74. Find the general solution of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also find the particular solution which passes through the lines $x = 1,\ y = 0$ **[12 Marks]**

75. Find the general solution of the partial differential equation: $(D^2 + DD' - 6D'^2)z = y\cos x$, where $D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y}$ **[12 Marks]**

76. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0,\ x = a,\ y = 0$ and $y = b$. The edges and $x = 0, x = a$ and $y = 0$ are kept at temperature zero while the edge $y = b$ is kept at 100^{0}C. **[30 Marks]**

77. Find complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method. **[15 Marks]**

78. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ canonical form. **[15 Marks]**

2007

79. (i) Form a partial differential equation by eliminating the function f from:

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$

(ii) Solve $2zx - px^2 - 2qxy + pq = 0$ **[6+6=12 Marks]**

80. Transform the equation $yz_x - xz_y = 0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution. **[12 Marks]**

81. Solve $u_{xx} + u_{yy} = 0$ in D where $D = \{(x, y) : 0 < x < a, 0 < y < b\}$ is a rectangle in a plane with the boundary conditions:

$$u(x,0) = 0,\ u(x,b) = 0,\ 0 \le x \le a$$
$$u(0,y) = g(y), u_x(a,y) = h(y),\ 0 \le y \le b.$$

[30 Marks]

82. Solve the equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the conditions: $u(0,t) = 0 = u(l,t)$, for all t and $u(x,0) = f(x)$ for all x in $[0,\ l]$ **[30 Marks]**

2006

83. Solve: $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$ **[12 Marks]**

84. Solve: $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$ **[12 Marks]**

85. The deflection of vibrating string of length l, is governed by the partial differential equation $u_{tt} = C^2\ u_{xx}$. The ends of the string are fixed at and $x = 0$ and l. The initial velocity is zero. The initial displacement is given by $u(x,0) = \begin{cases} \frac{x}{l}, & 0 < x < \frac{l}{2} \\ \frac{1}{l}(l - x), & \frac{l}{2} < x < l. \end{cases}$

Find the deflection of the string at any instant of time. **[30 Marks]**

86. Find the surface passing through the parabolas $z = 0,\ y^2 = 4ax$ and $z = 1,\ y^2 = -4ax$ and satisfying the equation $x\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial z}{\partial x} = 0$ **[15 Marks]**

87. Solve the equation $p^2 x + q^2 y = z,\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ **[15 Marks]**

2005

88. Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the coordinate planes. **[12 Marks]**

89. Find the particular integral of $x(y-z)p+y(z-x)q=z(x-y)$ which represents a surface passing through $x=y=z$ **[12 Marks]**

90. The ends A and B of a rod 20cm long have the temperature at 30^0C and 80^0C until steady state prevails. The temperatures of ends are changed to 40^0C and 60^0C respectively. Find the temperature distribution in the rod at time t. **[30 Marks]**

91. Obtain the general solution of $(D-3D'-2)^2 z = 2e^{2x}\sin(y+3x)$ where $D=\frac{\partial}{\partial x}$ and $D'=\frac{\partial}{\partial y}$ **[30 Marks]**

2004

92. Find the integral surface of the following partial differential equation: $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ **[12 Marks]**

93. Find the complete integral of the partial differential equation $(p^2+q^2)x=pz$ and deduce the solution which passes through the curve $x=0,\ z^2=4y$. **[12 Marks]**

94. Solve the partial differential equation: $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=(y-1)e^x$ **[15 Marks]**

95. A uniform string of length l, held tightly between $x=0$ and $x=l$ with no initial displacement, is struck at $x=a,\ 0<a<l,$ with velocity v_0. Find the displacement of the string at any time $t>0$ **[30 Marks]**

96. Using Charpit's method, find the complete solution of the partial differential equation $p^2x+q^2y=z$ **[15 Marks]**

2003

97. Find the general solution of $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y+\cos(2x+3y)$ **[12 Marks]**

98. Show that the differential equations of all cones which have their vertex at the origin are $px+qy=z$. Verify that $yz+zx+xy=0$ is a surface satisfying the above equation. **[12 Marks]**

99. Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}-3\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=xy+e^{x+2y}$ **[15 Marks]**

100. Solve the equation $p^2-q^2-2px-2qy+2xy=0$ using Charpit's method. Also find the singular

101. Find the deflection $u(x, t)$ of a vibrating string, stretched between fixed points $(0, 0)$ and $(3l, 0)$, corresponding to zero initial velocity and following initial deflection:

$$f(x) = \begin{cases} \dfrac{hx}{l} & \text{when } 0 \le x \le 1 \\ \dfrac{h(3l-2x)}{l} & \text{when } l \le x \le 2l \\ \dfrac{h(x-3l)}{l} & \text{when } 2l \le x \le 3l \end{cases}$$

Where h is a constant. **[15 Marks]**

2002

102. Find two complete integrals of the partial differential equation $x^2p^2 + y^2q^2 - 4 = 0$ **[12 Marks]**

103. Find the solution of the equation $z = \frac{1}{2}(p^2+q^2) + (p-x)(q-y)$ **[12 Marks]**

104. Frame the partial differential equation by eliminating the arbitrary constants a and b from $\log(az-1) = x + ay + b$ **[10 Marks]**

105. Find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = \frac{x}{2}, \ y = 0$ **[20 Marks]**

106. Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \ 0 < x < l, \ t > 0$

$u(0, t) = u(l, t) = 0$

$u(x, 0) = x(l-x), \ \ 0 \le x \le t.$ **[30 Marks]**

2001

107. Find the complete integral partial differential equation $2p^2q^2 + 3x^2y^2 = 8x^2q^2(x^2+y^2)$ **[12 Marks]**

108. Find the general integral of the equation $\{my(x+y) - nz^2\}\frac{\partial z}{\partial x} - \{lx(x+y) - nz^2\}\frac{\partial z}{\partial y} = (lx - my)z$

109. Prove that for the equation $z + px + qy - 1 - pqx^2y^2 = 0$ the characteristic strips are given by

$x(t) = \frac{1}{B+Ce^{-t}}, \ y(t) = \frac{1}{A+De^{-t}}, \ z(t) = E - (AC+BD)e^{-t}$

$p(t) = A(B+Ce^{-t})^2, \ q(t) = B(A+De^{-t})^2$ where A, B, C, D and E are arbitrary constants. Hence find the values of these arbitrary constants if the integral surface passes through the line $z = 0, \ x = y$ **[30 Marks]**

110. Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2+y^2+z^2) = C_1y^2$ **[10 Marks]**

111. Solve the equation $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x^2y^4$ by reducing it to the equation with constant coefficients. **[20 Marks]**

2000

112. Solve: $pq = x^m y^n z^{2l}$ **[12 Marks]**

113. Prove that if $x_1^3 + x_2^3 + x_3^3 = 1$ when $z = 0$, the solution of the equation $(S - x_1)p_1 + (S - x_2)p_2 + (S - x_3)p_3 = S - z$ can be given in the form $S^3\left\{(x_1 - z)^3 + (x_2 - z)^3 + (x_3 - z)^3\right\}^4 = \left(x_1 + x_2 + x_3 - 3z\right)^3$ where $S = x_1 + x_2 + x_3 + z$ and $p_i = \frac{\partial z}{\partial x_i},\ i = 1,2,3.$ **[12 Marks]**

114. Solve by Charpit's method the equation $p^2x(x-1) + 2pqxy + q^2y(y-1) - 2pxz - 2qyz + z^2 = 0$ **[15 Marks]**

115. Solve: $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{3x+4y}$. **[15 Marks]**

116. A tightly stretched string with fixed end points $x = 0,\ x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point x of it a velocity $kx(l - x)$, obtain at time t the displacement y at a distance x from the end $x = 0$ **[30 Marks]**

1999

117. Verify that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and find its primitive. **[20 Marks]**

118. Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$, c is constant, orthogonally and which passes through the circle $x^2 + y^2 = 1,\ z = 1$ **[20 Marks]**

119. Find the characteristics of the equation $pq = z$, and determine the integral surface which passes through the passes through the parabola $x = 0,\ y^2 = z.$ **[20 Marks]**

120. Use Charpit's method to find a complete integral to $p^2 + q^2 - 2px - 2qy + 1 = 0$ **[20 Marks]**

121. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-x}\cos y$ which $\to 0$ as $x \to \infty$ and has the value $\cos y$ when $x = 0$ **[20 Marks]**

122. One end of a string $(x = 0)$ is fixed, and the point $x = a$ is made to oscillate, so that at time t the displacement is $g(t)$. Show that the displacement $u(x,\ t)$ of the point x at time t is given by $u(x,\ t) = f(ct - x) - f(ct + x)$ where f is a function satisfying the relation $f(t + 2a) = f(t) - g\left(\frac{t + a}{c}\right)$ **[20 Marks]**

1998

123. Find the differential equation of the set of all right circular cones whose axes coincide with the z-axis **[20 Marks]**

124. Form the differential equation by eliminating a, b and c from $z = a(x + y) + b(x - y) + abt + c$ **[20 Marks]**

125. Solve $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = xyz$ **[20 Marks]**

126. Find the integral surface of the linear partial the differential equation $x(y^2 + z)\frac{\partial z}{\partial x} - y(x^2 + z)\frac{\partial z}{\partial y} = (x^2 - y^2)z$ through the straight line $x + y = 0,\ z = 1$ **[20 Marks]**

127. Use Charpit's method to find a complete integral of $2x\left[\left(z\dfrac{\partial z}{\partial y}\right)^2+1\right]=z\dfrac{\partial z}{\partial x}$ **[20 Marks]**

128. Find a real function $V(x,y)$, which reduces to zero when $y=0$ and satisfies the equation
$\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial y^2}=-4\pi(x^2+y^2)$ **[20 Marks]**

129. Apply Jacobi's method to find a complete integral of the equation
$2x\dfrac{\partial z}{\partial x_1}x_1x_3+3\dfrac{\partial z}{\partial x_2}x_3^2+x\left(\dfrac{\partial z}{\partial x_2}\right)^2 x\dfrac{\partial z}{\partial x_3}=0$ **[20 Marks]**

1997

130. (i) Find the differential equation of all surfaces of revolution having z- axis as the axis of rotation.
(ii) Form the differential equation by eliminating a and b from $z=(x^2+a)(y^2+b)$
[10+10=20 Marks]

131. Find the equation of surfaces satisfying $4yzp+q+2y=0$ and passing through
$y^2+z^2=1,\ x+z=2$ **[20 Marks]**

132. Solve: $(y+z)p+(z+x)q=x+y$ **[20 Marks]**

133. Use Charpit's method to find complete integral of $z^2(p^2z^2+q^2)=1$ **[20 Marks]**

134. Solve: $(D_x^{\ 3}-D_y^{\ 3})z=x^3y^3$ **[20 Marks]**

135. Apply Jacobi's method to find complete integral of $p_1^{\ 3}+p_2^{\ 2}+p_3=1$. Here

$p_1=\dfrac{\partial z}{\partial x_1},\ p_2=\dfrac{\partial z}{\partial x_2},\ p_3=\dfrac{\partial z}{\partial x_3}$ and z is a function of x_1,x_2,x_3. **[20 Marks]**

1996

136. (i) differential equation of all spheres of radius λ having their center in xy- plane
(ii) Form differential equation by eliminating f and g from $z=f(x^2-y)+g(x^2+y)$
[10+10=20 Marks]

137. Solve: $z^2(p^2+q^2+1)=C^2$ **[20 Marks]**

138. Find the integral surface of the equation $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ passing through the curve $xz=a^3,\ y=0$ **[20 Marks]**

139. Apply Charpit's method to find the complete integral of $z=px+qy+p^2+q^2$ **[20 Marks]**

140. Solve: $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=\cos mx\cos ny$ **[20 Marks]**

141. Find a surface passing through the lines $z=x=0$ and $z-1=x-y=0$ satisfying
$\dfrac{\partial^2 z}{\partial x^2}-4\dfrac{\partial^2 z}{\partial x\partial y}+4\dfrac{\partial^2 z}{\partial y^2}=0$ **[20 Marks]**

1995

142. In the context of a partial differential equation of the first order in three independent variables, define and illustrate the terms:
(i) The complete integral
(ii) The singular integral **[20 Marks]**

143. Find the general integral of $(y+z+w)\frac{\partial w}{\partial x}+(z+x+w)\frac{\partial w}{\partial y}+(x+y+w)\frac{\partial w}{\partial z}=x+y+z$ **[20 Marks]**

144. Obtain the differential equation of the surfaces which are the envelopes of a one-parameter family of planes. **[20 Marks]**

145. Explain in detail the Charpit's method of solving the nonlinear partial differential equation
$f\left(x,y,z,\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}\right)=0$ **[20 Marks]**

146. Solve $\frac{\partial z}{\partial x_1}\frac{\partial z}{\partial x_2}\frac{\partial z}{\partial x_3}=z^3x_1x_2x_3$ **[20 Marks]**

147. Solve $(D_x^{\,3}-7D_xD_y^{\,2}-6D_y^{\,3})z=\sin(x+2y)+e^{3x+y}$ **[20 Marks]**

1994

148. Find the differential equation of the family of all cones with vertex at $(2,-3,1)$ **[20 Marks]**

149. Find the integral surface of $x^2p+y^2q+z^2=0,\ p\equiv\frac{\partial z}{\partial x},\ q\equiv\frac{\partial z}{\partial y}$ which passes through the hyperbola
$xy=x+y,\ z=1$ **[20 Marks]**

150. Obtain a Complete Solution of $pq=x^my^nz^{2l}$ **[20 Marks]**

151. Use the Charpit's method to solve $16p^2z^2+9q^2z^2+4z^2-4=0$. Interpret geometrically the complete solution and mention the singular solution. **[20 Marks]**

152. Solve $(D^2+3DD'+2D'^2)z=x+y$, by expanding the particular integral in ascending powers of D, as well as in ascending powers of D'. **[20 Marks]**

153. Find a surface satisfying $(D^2+DD')z=0$ and touching the elliptic paraboloid $z=4x^2+y^2$ along its section by the plane $y=2x+1$. **[20 Marks]**

1993

154. Find the surface whose tangent planes cut off an intercept of constant length R from the axis of z. **[20 Marks]**

155. Solve $(x^3+3xy^2)p+(y^3+3x^2y)q=2(x^2+y^2)z$ **[20 Marks]**

156. Find the integral surface of the partial differential equation $(x-y)p+(y-x-z)q=z$ through the circle $z=1,\ x^2+y^2=1$ **[20 Marks]**

157. Using Charpit's method find the complete integral of $2xz-px^2-2qxy+pq=0$ **[20 Marks]**

158. Solve $r-s+2q-z=x^2y^2$ **[20 Marks]**

159. Find the general solution of $x^2r-y^2t+xp-yq=\log x$ **[20 Marks]**

1992

160. Solve:

$$(2x^2 - y^2 + z^2 - 2yz - zx - xy)p + (x^2 + 2y^2 + z^2 - yz - 2zx - xy)q = (x^2 + y^2 + 2z^2 - yz - zx - 2xy)$$

[20 Marks]

161. Find the complete integral of $(y-x)(qy-px) = (p-q)^2$ **[20 Marks]**

162. Use Charpit's method to solve $px + qy = z\sqrt{1+pq}$ **[20 Marks]**

163. Find the surface passing through the parabolas $z=0,\ y^2 = 4ax;\ z=1,\ y^2 = -4ax$ and satisfying the differential equation $xr + 2p = 0$ **[20 Marks]**

164. Solve: $r + s - 6t = y\cos x$ **[20 Marks]**

165. Solve: $\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 z}{\partial x \partial v} + \frac{\partial z}{\partial v} - z = \cos(x+2y) + e^y$ **[20 Marks]**