RAMANASRI IAS/IFoS Institute

Real Analysis

IFoS (IFS) Previous Year Questions (PYQ) from 2025 to 2009

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2025

1. Let S be a non-empty subset of $\mathbb{R}$, bounded below, and $T = \{-x : x \in S\}$. Prove that T is bounded above and $\sup T = -\inf S$. **[8 Marks]**

2. Prove that the series $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \cdots$ converges to $\frac{3}{2}\log 2$. **[8 Marks]**

3. Let $f_n(x) = nx(1-x)^n, x \in [0,1], n \in \mathbb{N}$. Show that (i) the sequence $\{f_n\}$ converges to a function f on $[0,1]$ and (ii) f is integrable on $[0,1]$ and $\lim_{n\to\infty} \int_0^1 f_n = \int_0^1 f$, but still the convergence of the sequence $\{f_n\}$ is not uniform. **[5+10 Marks]**

4. Evaluate **[10 Marks]**

$$\iint_E \frac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}}\, dx\, dy,$$

the field of integration E being the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

2024

5. Show that the series **[8 Marks]**

$$\frac{1}{(1+a)^p} - \frac{1}{(2+a)^p} + \frac{1}{(3+a)^p} - \cdots, \qquad a > 0,$$

is (i) absolutely convergent if $p > 1$ and (ii) conditionally convergent if $0 < p \le 1$.

6. If $f'(x) = (x-a)^{2n}(x-b)^{2m+1}$, where m, n are positive integers, show that f has neither a maximum nor a minimum at a and f has a local minimum at b. **[8 Marks]**

7. Examine the convergence of the improper integral $\int_0^\infty \frac{x^{p-1}}{1+x}\, dx$ and hence evaluate it. **[15 Marks]**

8. Evaluate $\lim_{x\to 0} \frac{\int_0^{x^2} e^{\sqrt{1+t}}\, dt}{x^2}$. **[10 Marks]**

2023

9. Find the relative extrema of the function $f(x,y) = 4y^3 + x^2 - 12y^2 - 36y + 2$. **[8 Marks]**

10. Prove that in the interval $0 < x < 1$, the function $f(x) = x^2$ is uniformly continuous while $f(x) = \frac{1}{x}$ is not uniformly continuous. **[8 Marks]**

11. Find the volume of the region above the xy-plane bounded by the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = a^2$. Also prove that $\lim_{M\to\infty} \int_0^M \frac{dx}{x^4+4} = \frac{\pi}{8}$. **[7+8 Marks]**

12. If $u_n(x), n = 1, 2, 3, \ldots$, are continuous in $[a,b]$ and if $\sum u_n(x)$ converges uniformly to the sum $S(x)$ in $[a,b]$, then prove that $S(x)$ is continuous in $[a,b]$. Prove that an absolutely convergent series is convergent. Show that $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is conditionally convergent. **[5+5 Marks]**

2022

13. Let $\mathbb{R}$ denote the set of real numbers and $\mathbb{Q}$ denote the set of rational numbers. If $x \in \mathbb{R}$, $x > 0$ and $y \in \mathbb{R}$, then show that there exists a positive integer n such that $nx > y$. Use it to show that if $x < y$, then there exists $p \in \mathbb{Q}$ such that $x < p < y$. **[8 Marks]**

14. Suppose $f : [a, b] \to \mathbb{R}$ is a continuous function. Then show that f is Riemann integrable on $[a, b]$. **[8 Marks]**

15. Suppose $\{f_n\}$ is a sequence of functions defined on $[a, b]$ and $\lim_{n\to\infty} f_n(x) = f(x)$, $x \in [a, b]$. Put **[7+8 Marks]**

$$M_n = \sup_{x\in[a,b]} |f_n(x) - f(x)|.$$

Then show that (i) f_n converges to f uniformly on $[a, b]$ if and only if $M_n \to 0$ as $n \to \infty$; (ii) if $|f_n(x)| \le M_n$, $(x \in [a, b],\ n = 1, 2, \dots)$, then $\sum_{n=1}^{\infty} f_n$ converges uniformly on $[a, b]$ if $\sum_{n=1}^{\infty} M_n$ converges.

16. Prove that every bounded and monotonically increasing sequence is convergent and converges to lub (least upper bound) of the sequence. If $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$, $\forall n \in \mathbb{N}$, then using Cauchy criterion for convergence of the sequence, show that $\{a_n\}$ is not convergent. **[5+5 Marks]**

2021

17. Apply Cauchy's Principle of Convergence to prove that the sequence $\{f_n\}$ defined by **[8 Marks]**

$$f_n = 1 + \frac{1}{4} + \frac{1}{7} + \cdots + \frac{1}{3n-2}$$

is not convergent.

18. Find $\frac{dy}{dx}$, when **[8 Marks]**

$$f(x, y) = \log(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right) = 0,$$

on using derivatives of Implicit Functions.

19. Examine the convergence of $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ and find its value, if possible. **[15 Marks]**

20. Examine the existence of maxima and minima of the function $u(x, y) = xy + \frac{8}{x} + \frac{8}{y}$. **[10 Marks]**