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Vector Analysis

Previous year Questions

from 2025 to 1992

2025

1. If $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$, then show that $\text{grad}\, u$, $\text{grad}\, v$ and $\text{grad}\, w$ are coplanar. **[10 Marks]**

2. Find the absolute value of the directional derivative of $\phi(x, y, z) = x^2 y^2 z^2$ at the point $(1, 1, -1)$ in the direction of the tangent to the curve $x = e^t$, $y = 2\sin t + 1$, $z = t - \cos t$, at $t = 0$. **[10 Marks]**

3. If $\nabla \cdot \vec{E} = 0$, $\nabla \cdot \vec{H} = 0$, $\nabla \times \vec{E} = -\dfrac{\partial \vec{H}}{\partial t}$ and $\nabla \times \vec{H} = \dfrac{\partial \vec{E}}{\partial t}$, then show that $\nabla^2 \vec{H} = \dfrac{\partial^2 \vec{H}}{\partial t^2}$ and $\nabla^2 \vec{E} = \dfrac{\partial^2 \vec{E}}{\partial t^2}$. **[10 Marks]**

4. Verify Green's theorem in the plane for $\oint_C \left[(xy + y^2)\, dx + x^2\, dy\right]$, where C is the boundary of the region bounded by the curves $y = x$ and $y = x^2$. **[15 Marks]**

5. Verify Gauss' divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$, taken over the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. **[15 Marks]**

2024

6. At any time t (in seconds), the coterminous edges of a variable parallelepiped are represented by the vectors $\vec{\alpha} = t\hat{i} + (t+1)\hat{j} + (2t+1)\hat{k}$, $\vec{\beta} = 2t\hat{i} + (3t-1)\hat{j} + t\hat{k}$ and $\vec{\gamma} = \hat{i} + 3t\hat{j} + \hat{k}$. What is the rate of change of the vectorial area of the parallelogram whose coterminous edges are $\vec{\alpha}$ and $\vec{\gamma}$? Also find the rate of change of the volume of the parallelepiped at $t = 1$ second. **[10 Marks]**

7. Let C be a plane curve $\vec{r}(t) = f(t)\hat{i} + g(t)\hat{j}$, where f and g have second-order derivatives. Show that the curvature at a point is given by **[5 Marks]**

$$\kappa = \frac{|f'(t)g''(t) - g'(t)f''(t)|}{\{[f'(t)]^2 + [g'(t)]^2\}^{3/2}}.$$

   What is the value of torsion τ at any point of this curve?

8. Show that the principal normals at two consecutive points of a curve do not intersect unless torsion τ is zero. **[5 Marks]**

9. State Stokes' theorem and verify it for the vector field $\vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$ over the surface S, which is the upwardly oriented part of the cylinder $z = 1 - x^2$, for $0 \le x \le 1$, $-2 \le y \le 2$. **[20 Marks]**

10. Using Gauss divergence theorem, evaluate $\iint_S \left(y^2\hat{i} + xz^3\hat{j} + (z-1)^2\hat{k}\right) \cdot \hat{n}\, dS$ over the region bounded by the cylinder $x^2 + y^2 = 16$ and the planes $z = 1$ and $z = 5$. **[15 Marks]**

2023

11. If $\vec{a} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j} + \theta\hat{k}$, $\vec{b} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j} - 3\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} - 3\hat{k}$, then find the values of the derivative of the vector function $\vec{a} \times (\vec{b} \times \vec{c})$ with respect to θ at $\theta = \frac{\pi}{2}$ and $\theta = \pi$. **[10 Marks]**

12. Evaluate the integral $\iint_S \left(3y^2z^2\hat{i} + 4z^2x^2\hat{j} + z^2y^2\hat{k}\right) \cdot \hat{n}\,dS$, where S is the upper part of the surface $4x^2 + 4y^2 + 4z^2 = 1$ above the plane $z = 0$ and bounded by the xy-plane. Hence, verify Gauss-Divergence theorem. **[20 Marks]**

13. If the tangent to a curve makes a constant angle θ with a fixed line, then prove that the ratio of radius of torsion to radius of curvature is proportional to $\tan\theta$. Further prove that if this ratio is constant, then the tangent makes a constant angle with a fixed direction. **[15 Marks]**

14. For a scalar point function ϕ and vector point function $\vec{f}$, prove the identity $\nabla \cdot (\phi\vec{f}) = \nabla\phi \cdot \vec{f} + \phi(\nabla \cdot \vec{f})$. Also find the value of $\nabla \cdot \left(\frac{f(r)}{r}\vec{r}\right)$ and then verify the stated identity. **[15 Marks]**

2022

15. Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Also find ϕ such that $\vec{A} = \nabla\phi$. **[10 Marks]**

16. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$, where C is the boundary curve of the region defined by $x = 0$, $y = 0$, $x + y = 1$. **[15 Marks]**

17. Verify Stokes' theorem for $\vec{F} = x\hat{i} + z^2\hat{j} + y^2\hat{k}$ over the plane surface $x + y + z = 1$ lying in the first octant. **[20 Marks]**

18. Using Gauss' divergence theorem, evaluate $\iint_S \vec{F} \cdot \hat{n}\,dS$, where $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ and S is the cylinder formed by the surfaces $z = 0$, $z = 1$, $x^2 + y^2 = 4$. **[15 Marks]**

2021

19. Show that $\nabla^2 \left[\nabla \cdot \left(\frac{\vec{r}}{r}\right)\right] = \frac{2}{r^4}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. **[10 Marks]**

20. Evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where C is an arbitrary closed curve in the xy-plane and $\vec{F} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$. **[15 Marks]**

21. Verify Gauss divergence theorem for $\vec{F} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$. **[20 Marks]**

22. Using Stokes' theorem, evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$, where $\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xy + z^2)\hat{k}$ and S is the surface of the paraboloid $z = 4 - (x^2 + y^2)$ above the xy-plane. Here, $\hat{n}$ is the unit outward normal vector on S. **[15 Marks]**

2020

23. For what value of a, b, c is the vector field $\overline{V} = (-4x - 3y + az)i + (bx + 3y + 5z)j + (4x + cy + 3z)k$ irrotational? Hence, express $\overline{V}$ as the gradient of a scalar function ϕ determine ϕ **[10 Marks]**

24. For the vector function $\overline{A}$ where $\overline{A} = (3x^2 + 6y)i - 14yzj + 20xz^2k$, calculate $\int_C \overline{A}.d\overline{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following paths:
(i) $x = t, y = t^2, z = t^3$
(ii) Straight lines joining $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$
(iii) Straight line joining $(0,0,0)$ to $(1,1,1)$ is the result same in all the cases? Explain the reason. **[15 Marks]**

25. Verify the stokes theorem for the vector field $\overline{F} = xyi + yzj + xzk$ on the surface S which is the part of the cylinder $z = 1 - x^2$ for $0 \le x \le 1, -2 \le y \le 2$; S is oriented upwards. **[20 Marks]**

26. Evaluate the surface integral $\iint_s \nabla \times \overline{F}.nds$ for $\overline{F} = yi + (x - 2xz)j - xyk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane **[15 Marks]**

2019

27. Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t^2, y = t^2, z = t^3$ at the point $(1,1,1)$ **[10 Marks]**

28. Find the circulation of $\vec{F}$ round the curve C where $\vec{F} = (2x + y^2)i(3y - 4x)j$ and C is the curve $y^2 = x$ from $(0,0)$ to $(1,1)$ and the curve $y = x^2$ from $(1,1)$ to **[15 Marks]**

29. Find the radius of curvature and radius of torsion of the helix $x = a\cos u,\ \ y = a\sin u, z = au\tan\alpha$ **[15 Marks]**

30. State Gauss divergence theorem. Verify this theorem for $\vec{F} = 4xi - 2y^2 j + z^2 k$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$ **[15 Marks]**

31. Evaluation by Stoke's theorem $\oint_C e^x dx + 2ydy - dz$ where C is the curve $x^2 + y^2 = 4,\ \ z = 2$. **[05 Marks]**

2018

32. Find the angle between the tangent at a general point of the curve whose equations are $x = 3t, y = 3t^2, z = 3t^3$ and the line $y = z - x = 0$ **[10 Marks]**

33. Let $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$ .Show that curl (curl $\vec{v}$) =grad (div $\vec{v}$) $-\nabla^2\vec{v}$. **[12 Marks]**

34. Evaluate the line integral $\int_C -y^3dx + x^3dy + z^3dz$ using stokes theorem. Here C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. The orientation on C corresponds to counterclockwise motion in the xy-plane. **[13 Marks]**

35. Let $\vec{F} = xy^2\vec{i} + (y + x)\vec{j}$ Integrate $(\nabla \times \vec{F}).\vec{k}$ over the region in the first quadrant bounded by the curves $y = x^2$ and $y = x$ using Green's theorem. **[13 Marks]**

36. Find the curvature and torsion of the curve $\vec{r} = a(u - \sin u)\vec{i} + a(1 - \cos u)\vec{j} + bu\vec{k}$ **[12 Marks]**

37. If S is the surface of the sphere $x^2+y^2+z^2=a^2$, then evaluate $\iint_S \left[(x+z)dydz+(y+z)dzdx+(x+y)dxdy\right]$ using gauss' divergence theorem. **[12 Marks]**

2017

38. For what values of the constant a, b and c the vector $\bar{V}=(x+y+az)\hat{i}+(bx+2y-z)j+(-x+cy+2z)k$ is irrotational. Find the divergence in cylindrical coordinates of the vector with these values. **[10 Marks]**

39. The position vector of a moving point at time t is $\bar{r}=\sin t\ \hat{i}+\cos 2t\ j+(t^2+2t)\ k$. Find the components of acceleration $\bar{a}$ in the direction parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time $t=0$. **[10 Marks]**

40. Find the curvature vector and its magnitude at any point $\bar{r}=(\theta)$ of the curve $\bar{r}=(a\cos\theta, a\sin\theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2+y^2-z^2=a^2$. **[16 Marks]**

41. Evaluate the integral $\iint_S \vec{F}.nds$ where $\vec{F}=3xy^2\hat{i}+(yx^2-y^3)j+3zx^2K$ and S is a surface of the cylinder $y^2+z^2\le 4,\ -3\le x\le 3$ using divergence theorem. **[9 Marks]**

42. Using Green theorem evaluate the $\int_C F(\vec{r}).d\vec{r}$ counterclockwise where $F(\vec{r})=(x^2+y^2)\hat{i}+(x^2-y^2)j$ and $d\vec{r}=x\hat{i}+dy\,j$ and the curve C is the boundary off the region $R=\left\{(x,y)\middle|\ 1\le y\le 2-x^2\right\}$. **[8 Marks]**

2016

43. Prove that the vector $\vec{a}=3\vec{i}+\hat{j}-2\hat{k}, \vec{b}=-\hat{i}+3\hat{j}+4\hat{k}, \vec{c}=4\hat{i}-2\hat{j}-6\hat{k}$ can from the sides of a triangle find the length of the medians of the triangle **[10 marks]**

44. Find $f(r)$ such that $\nabla f=\frac{\vec{r}}{r^5}$ and $f(1)=0$ **[10 marks]**

45. Prove that $\oint_C fd\vec{r}=\iint_S d\vec{S}\times\nabla f$ **[10 marks]**

46. For the of cardioid $r=a(1+\cos\theta)$ show that the square of the radius of curvature at any point (r,θ) is proportion to r. Also find the radius of curvature if $\theta=0,\frac{\pi}{4},\frac{\pi}{2}$. **[15 marks]**

2015

47. Find the angle between the surfaces $x^2+y^2+z^2-9=0$ and $z=x^2+y^2-3$ at $(2,-1,2)$ **[10 Marks]**

48. A vector field is given by $\vec{F}=(x^2+xy^2)\hat{i}+(y^2+x^2y)\hat{j}$. Verify that the field is irrotational or not. Find the scalar potential. **[12 Marks]**

49. Evaluate $\int_C e^{-x}(\sin y dx+\cos y dy)$, where C is the rectangle with vertices $(0,0)(\pi,0),\left(\pi,\frac{\pi}{2}\right),\left(0,\frac{\pi}{2}\right)$ **[12 Marks]**

2014

50. Find the curvature vector at any point of the curve $\bar{r}(t) = t\cos t\hat{i} + t\sin t\hat{j},\ 0 \le t \le 2\pi$. Give its magnitude also. **[10 Marks]**

51. Evaluate by Stokes' theorem $\int_{\Gamma}\left(ydx + zdy + xdz\right)$, where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y = 2a$ starting from $(2a,0,0)$ and then going below the z-plane. **(20 Marks]**

2013

52. Show the curve $\vec{x}(t) = t\hat{i} + \left(\frac{1+t}{t}\right)\hat{j} + \left(\frac{1-t^2}{t}\right)\hat{k}$ lies in a plane. **[10 Marks]**

53. Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x,y,z) from the origin, n being a constant and ∇^2 being the Laplace operator **[10 Marks]**

54. A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$ **[10 Marks]**

55. By using Divergence Theorem of Gauss, evaluate the surface integral $\iint\left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-\frac{1}{2}}\,dS$, where S is the surface e of the ellipsoid $ax^2 + by^2 + cz^2 = 1,\ a,b$ and c being all positive constants. **[15 Marks]**

56. Use Stokes' theorem to evaluate the line integral $\int_C(-y^3dx + x^3dy - z^3dz)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$ **[15 Marks]**

2012

57. If $\vec{A} = x^2yz\vec{i} - 2xz^3\vec{j} + xz^2\vec{k},\ \vec{B} = 2z\vec{i} + y\vec{j} - x^2\vec{k}$ find the value of $\frac{\partial^2}{\partial x\partial y}(\vec{A} + \vec{B})$ at $(1,0,-2)$ **[12 Marks]**

58. Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$. Show that the curvature and torsion are equal for this curve. **[20 Marks]**

59. Verify Green's theorem in the plane for $\oint_C\left[xy + y^2dx + x^2dy\right]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ **[20 Marks]**

60. If $\vec{F} = y\vec{i} + \left(x - 2xz\right)\vec{j} - xy\vec{k},$, evaluate $\iint_S(\vec{\nabla}\times\vec{F}).\vec{n}d\vec{s}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. **[20 Marks]**

2011

61. For two vectors $\vec{a}$ and $\vec{b}$ give respectively by $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin 5t\hat{i} - \cos t\hat{j}$ determine:(i) $\frac{d}{dt}(\vec{a}.\vec{b})$ and (ii) $\frac{d}{dt}(\vec{a} \times \vec{b})$ **[10 Marks]**

62. If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f} = grad v$, find the value of $\vec{f} curl \vec{f}$ **[10 Marks]**

63. Examine whether the vectors $\nabla u, \nabla u$ and ∇w are coplanar, where u, v and w are the scalar functions defined by:

$$u = x + y + z,$$
$$v = x^2 + y^2 + z^2$$
$$\text{and } w = yz + zx + xy$$

[15 Marks]

64. If $\vec{u} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$ calculate the double integral $\iint(\nabla \times \vec{u})d\vec{s}$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2, z \geq 0$ **[15 Marks]**

65. If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector. $r^n\vec{r}$ is (i) irrotational, (ii) solenoidal **[15 Marks]**

66. Verify Gauss' Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube $0 \leq x, y, z \leq 1$. **[15 Marks]**

2010

67. Find the directional derivative of $f(x, y) = x^2y^3 + xy$ at the point (2,1) in the direction of a unit vector which makes an angle or $\frac{\pi}{3}$ with the x-axis. **[12 Marks]**

68. Show that the vector field defined by the vector function $\vec{v} = xyz(yz\vec{i} + xy\vec{j} + xy\vec{k})$ is conservative. **[12 Marks]**

69. Prove that $div(f\vec{V}) = f(div\vec{V}) + (grad.f)\vec{V}$ where f is a scalar function. **[20 Marks]**

70. Use the divergence theorem to evaluate $\iint_s \vec{V}\vec{n}dA$ where $\vec{V} = x^2z\vec{i}y\vec{j} - xz^2\vec{k}$ and S is he boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$. **[20 Marks]**

71. Verify Green's theorem for $e^{-x}\sin ydx + e^{-x}\cos y$ by the path of integration being the boundary of the square whose vertices are $(0,0), \left(\frac{\pi}{2,0}\right)\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\left(\frac{0,\pi}{2}\right)$ **[20 Marks]**

2009

72. Show that $div(grad r^n) = n(n+1)r^{n-2}$ where $r = \sqrt{x^2 + y^2 + z^2}$. **[12 Marks]**

73. Find the directional derivative of $(i) 4xz^3 - 3x^2y^2z^2$ (i) at $(2,-1,2)$ along z-axis $(ii) - x^2yz + 4xz^2$ at $(1,-2,1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{\kappa}$. **[6+6=12 Marks]**

74. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z=0$ under the field of force of given by $\overline{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}(3x-2y+4z)\hat{k}$. **[20 Marks]**

75. Using divergence theorem, evaluate $\iint_s \overline{A}.d\overline{S}$ where $\overline{A}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$ **[20 Marks]**

76. Find the value of $\iint_s (\vec{\nabla}\times\vec{f}).d\vec{s}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane $z=0$,when $\vec{F}=(y^2+z^2-x^2)\hat{i}+(z^2+x^2-y^2)\hat{j}+(x^2+y^2-z^2)\hat{k}$ **[20 Marks]**

2008

77. Find the constants a and b so that the surface $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point $(1,-1,2)$. **[12 Marks]**

78. Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from $(1-2,1)$ to $(3,1,4)$. **[12 Marks]**

79. Prove that $\nabla^2 f(x)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$ where $r=(x^2+y^2+z^2)^{\frac{1}{2}}$. Hence find $f(x)$ such that $\nabla^2 f(r)=0$. **[15 Marks]**

80. Show that for the space curve $x=t, y=t^2, z=\frac{2}{3}t^3$ the curvature and torsion are same at every point. **[15 Marks]**

81. Evaluate $\int_c \vec{A}d\vec{r}$ along the curve $x^2+y^2=1, z=1$ form $(0,1,1)$ to $(1,0,1)$ if $\vec{A}=(yz+2x)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}$. **[15 Marks]**

82. Evaluate $\iint_s \vec{F}\hat{n}d$ were $\vec{A}=(yz+2x)\hat{i}+xz\hat{j}+(xy+2z)\hat{k}, \iint_s \vec{F}\hat{n}ds$ and S is the surface of the cylinder bounded by $x^2+y^2=4z=0$ and $z=3$ **[15 Marks]**

2007

83. If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r=|\vec{r}|$ determined grad(r_{-1}) in terms of $\hat{r}$ and r. **[12 Marks]**

84. Find the curvature and torsion at any point of the curve $x=a\cos 2t, y=a\sin 2t, z=2a\sin t$. **[12 Marks]**

85. For any constant vector, show that the vector $\vec{a}$ represented by curl$(\vec{a}\times\vec{r})$ is always parallel to the vector $\vec{a},\vec{r}$ being the position vector of a point (x,y,z) measured from the origin. **[15 Marks]**

86. If $\vec{r}=x\hat{i}+y\hat{j}+x\hat{k}$ find the value(s) of in order that $r^n\vec{r}$ may be (i) solenoidal (ii) irrotational **[15 Marks]**

87. Determine $\int_c (ydx + zdy + xdz)$ by using Stoke's theorem, where C is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2, x + y = 2a$ that starts from the point $(2a, 0, 0)$ goes at first below the z-plane. **[15 Marks]**

2006

88. Find the values of constants a, b and c so that the directional derivative of the function $f = axy^2 + byz + cz^2x^2$ at the point $(1, 2, -1)$ has maximum magnitude 64 in the direction parallel to z-axis. **[12 Marks]**

89. If $\overline{A} = 2\overline{i} + \overline{K}, \overline{B} = \overline{i} + \overline{j} + \overline{K}, \overline{C} = 4\overline{i} - 3\overline{j} - 7\overline{K}$ determine a vector $\overline{R}$ satisfying the vector equation $\overline{R} \times \overline{B} = \overline{C} \times \overline{B}$ & $\overline{R}.\overline{A} = 0$ **[15 Marks]**

90. Prove that $r^n \overline{r}$ is an irrotational vector for any value o f n but is solenoidal only if $n + 3 = 0$ **[15Marks]**

91. If the unit tangent vector $\overline{t}$ and binormal $\overline{b}$ make angles ϕ and ϕ respectively with a constant unit vector $\overline{a}$ prove that $\frac{\sin\theta}{\sin\phi}.\frac{d\theta}{d\phi}. = -\frac{k}{\tau}$. **[15 Marks]**

92. Verify Stokes' theorem for the function $\overline{F} = x^2\hat{i} - xy\hat{j}$ integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a$ and $y = a, a > 0$. **[15 Marks]**

2005

93. Show that the volume of the tetrahedron $ABCD$ is $\frac{1}{6}\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\overrightarrow{AD}$ Hence find the volume of the tetrahedron with vertices $(2,2,2), (2,0,0), (0,2,0)$ and $(0,0,2)$ **[12 Marks]**

94. Prove that the curl of a vector field is independent of the choice of coordinates **[12 Marks]**

95. The parametric equation of a circular helix is $r = a\cos u\hat{i} + a\sin u\hat{j} + cu\hat{k}$ where c is a constant and u is a parameter. Find the unit tangent vector $\hat{t}$ at the point u and the arc length measured form $u = 0$ Also find $\frac{d\hat{t}}{ds}$ where s is the arc length. **[15 Marks]**

96. Show that $\text{curl}\left(k \times \text{grad}\frac{1}{r}\right) + \text{grad}\left(k.\text{grad}\frac{1}{r}\right) = 0$ where r is the distance from the origin and k is the unit vector in the direction OZ **[15 Marks]**

97. Find the curvature and the torsion of the space curve **[15 Marks]**

98. Evaluate by Gauss divergence theorem, where S is the surface of the cylinder bounded by and **[15 Marks]**

2004

99. Show that if $\overline{A}$ and $\overline{B}$ are irrotational, then $\overline{A} \times \overline{B}$ is solenoidal. **[12 Marks]**

100. Show that the Frenet-Serret formulae can be written in the form $\frac{d\overline{T}}{ds} = \overline{\omega} \times \overline{T}, \frac{d\overline{N}}{ds} = \overline{\omega} \times \overline{N}$ & $\frac{d\overline{B}}{dx} = \overline{\omega} \times \overline{B}$, where $\overline{\omega} = \tau\overline{T} + k\overline{B}$. **[12 Marks]**

101. Prove the identity $\nabla\left(\overline{A}.\overline{B}\right) = \left(\overline{B}.\nabla\right)\overline{A} + \left(\overline{A}.\nabla\right)\overline{B} + \overline{B} \times \left(\nabla \times \overline{A}\right) + \overline{A} \times \left(\nabla \times \overline{B}\right)$ **[15 Marks]**

102. Derive the identity $\iiint_v \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right)dV = \iint_s \left(\phi\nabla\psi - \psi\nabla\phi\right)\hat{n}dS$ where V is the volume bounded by the closed surface S. **[15 Marks]**

103. Verify Stokes' theorem for $\hat{f} = (2x-y)\hat{i} - yz^2\hat{j}zk\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. **[15 Marks]**

2003

104. Show that if a' b' and c' are the reciprocals of the non-coplanar vectors a, b and c , then any vector r may be expressed as $r=(r.a')a+(r.b')b+(r.c')c.$ **[12 Marks]**

105. Prove that the divergence of a vector field is invariant w, r, to co-ordinate transformations. **[12 Marks]**

106. Let the position vector of a particle moving on a plane curve be $r(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions. **[15 Marks]**

107. Prove the identity $\nabla A^2 = 2(A.\nabla)A + 2A\times(\nabla\times A)$ where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ **[15 Marks]**

108. Find the radii of curvature and torsion at a point of intersection of the surface $x^2 - y^2 = c^2, y = x\tanh\left(\frac{z}{c}\right).$ **[15 Marks]**

109. Evaluate $\iint_s curtA.ds$ where S is the open surface $x^2+y^2-4x+4z=0, z\ge 0$ and $A=\left(y^2+z^2-x^2\right)\hat{i}+\left(2z^2+x^2-y^2\right)\hat{j}+\left(x^2+y^2-3z^2\right)\hat{k}$. **[15 Marks]**

2002

110. Let $\overline{R}$ be the unit vector along the vector $\overline{r}(t)$ Show that $\overline{R}\times\frac{\overline{dR}}{dt} = \frac{\overline{r}}{r^2}\times\frac{\overline{dr}}{dt}$ where $r=\left|\overline{r}\right|$ **[12 Marks]**

111. Find the curvature k for the space curve $x = a\cos\theta, y = a\sin\theta, z = a\theta\tan\alpha$ **15 Marks]**

112. Show that $\left(cult\overline{v}\right) = grad\left(div\overline{v}\right) - \nabla^2\overline{v}$. **[15 Marks]**

113. Let D be a closed and bounded region having boundary S. Further, let f is a scalar function having second partial derivatives defined on it. Show that $\iint_s (fgradf).\hat{n}ds = \iiint_v \left[\left|gradf\right|^2 + f\nabla^2 f\right]dv$ Hence $\iint_s (fgradf).\hat{n}ds$ or otherwise evaluate for $f = 2x+y+2z$ over $s = x^2+y^2+z^2 = 4$ **[15 Marks]**

114. Find the values of constants a, b and c such that the maximum value of directional derivative of $f = axy^2 + byz + cx^2z^2$ at $(1,-1,1)$ is in the direction parallel to y-axis and has magnitude 6 **[15 Marks]**

2001

115. Find the length of the arc of the twisted curve $r=\left(3t, 3t^2, 2t^3\right)$ from the point $t=0$ to the point $t=1$ Find also the unit tangent t, unit normal n and the unit binormal b at $t=1$. **[12 Marks]**

116. Show that $curl\frac{a\times r}{r^3} = -\frac{a}{r^3} + \frac{3r}{r^5}(a.r)$ where a is constant vector. **[12 Marks]**

117. Find the directional derivative of $f = x^2yz^3$ along $x = e^{-t}, y = 1+2\sin t, z = t-\cos t$ at $t = 0$ **[15 Marks]**

118. Show that the vector field defined by $F = 2xyz^3 i + x^2z^3 j + 3x^2yz^2 k$ is irrotational. Find also the scalar u such that $F = grad\ u$ **[15 Marks]**

119. Verify Gauss' divergence theorem of $A = (4x, -2y^2, z^2)$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$. **[15 Marks]**

2000

120. In what direction from the point $(-1,1,1)$ is the directional derivative $f = x^2yz^3$ of a maximum? Compute its magnitude. **[12 Marks]**

121. Show that the covariant derivatives of the fundamental metric tensors $g_{ij}.g^{ij}, \delta^i{}_j$ Vanish
(ii) Show that simultaneity is relative in special relativity theory. **[6+6=12 Marks]**

122. Show that
$(i)(A+B).(B+C)\times(C+A) = 2A.B\times C$
$(ii)\nabla\times(A\times B) = (B.\nabla)A - B(\nabla.A) - (A.\nabla)B + A(\nabla.B)$ **[7+8=15 Marks]**

123. Evaluate $\iint_S F.Nds$ where $F = 2xyi + yz^2 j + xzk$ and S is the surface of the parallelepiped bounded by $x = 0, y = 0, z = 0.x = 2.y = 1$ and $z = 3$ **[15 Marks]**

124. If g_{ij} and γ_{ij} are two metric tensors defined at a point and $\left\{ {l \atop ij} \right\}$ and $\left[{l \atop ij} \right.$ are the corresponding Christoffel symbols of the second kind, then prove that $\left\{ {l \atop ij} \right\} - \left[{l \atop ij} \right.$ is a mixed tensor of the type $A^l{}_{ij}$ **[15 Marks]**

125. Establish the formula $E = mc^2$ the symbols have their usual meaning. **[15 Marks]**

1999

126. If $\bar{a}, \bar{b}\bar{c}$ are the position vectors of A, B, C prove that $\bar{a}\times\bar{b} + \bar{b} + \bar{c} + \bar{c}\times\bar{a}$ is vector perpendicular to the plane ABC **[20 Marks]**

127. If $\bar{f} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $\nabla\times\bar{F}$. **[20 Marks]**

128. Evaluate $\int_c (e^{-x}\sin y dx + e^{-x}\cos y dy)$ (by Green's theorem), where C is the rectangle whose vertices are $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$ **[20 Marks]**

1998

129. If r_1 and r_2 are the vectors joining the fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ respectively to a variable point $P(x, y, z)$ then the values of grad $(r_1.r_2)$ and curl $(r_1\times r_2)$. **[20 Marks]**

130. Show that $(a\times b)\times c = a\times(a\times b)$ if either $b = 0$ (or any other vector is 0) or c is collinear with a or b is orthogonal to a and c (both) **[20 Marks]**

131. Prove that $\left\{ {i \atop ik} \right\} = \frac{\partial}{\partial x_k}\left(\log\sqrt{g}\right)$. **[20 Marks]**

1997

132. Prove that if $\vec{A},\vec{B}$ and $\vec{C}$ are there given non-coplanar vectors $\vec{F}$ then any vector can be put in the form $F=\alpha\vec{B}\times\vec{C}+\beta\vec{C}\times\vec{A}+\gamma\vec{A}\times\vec{B}$ for given determine α,β,γ. **[20 Marks]**

133. Verify Gauss theorem for $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by $x^2+y^2=4, z=0$ and $z=3$ **[20 Marks]**

134. Prove that the decomposition of a tensor into a symmetric and an anti-symmetric part is unique. Further show that the contracted product $S_{ij}T_{ij}$ of a tensor T_{ij} with a symmetric tensor S_{ij} is independent of the anti-symmetric part of T_{ij}. **[20 Marks]**

1996

135. State and prove 'Quotient law' of tensors **[20 Marks]**

136. If $x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$ show that

$(i)\vec{r}\times gradf(r)=0$

$(ii)div(r^n\vec{r})=(n+3)r^n$ **[20 Marks]**

137. Verify Gauss's divergence theorem for $\vec{F}=xyx\hat{i}+z^2\hat{j}+2yz\hat{k}$ on the tetrahedron $x=y=z=,x+y+z=1$. **[20 Marks]**

1995

138. Consider a physical entity that is specified by twenty-seven numbers A_{ijk} in given coordinate system. In the transition to anther coordinates system of this kind. Let $A_{ijk}B_{jk}$ transform as a vector for any choice of the anti-symmetric tensor. Prove that the quantities $A_{ijk}-A_{ijk}$ are the components of a tensor B_{jk} of third order. Is the component of tensor? Give reasons for your answer **[20 Marks]**

139. Let the reason V be bounded by the smooth surface S and let n denote outward drawn unit normal vector at a point on S. If ϕ is harmonic in V, show that $\int_s \frac{\partial\phi}{\partial n}ds=0$ **[20 Marks]**

140. In the vector field $u(x)$ let there exists a surface $curl\upsilon$ on which $\upsilon=0$. Show that, at an arbitrary point of this surface $curl\upsilon$ is tangential to the surface or vanishes. **[20 Marks]**

1994

141. Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n=-3$. **[20 Marks]**

142. If $\vec{F}=y\vec{i}+(x-2xz)\vec{j}-xy\vec{k}$ evaluate $\iint_s\left(\Delta\times\vec{F}\right)\vec{n}ds$ Where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy plane. **[20 Marks]**

143. Prove that $\left\{\begin{matrix} i \\ ik \end{matrix}\right\}=\frac{\partial}{\partial x}\left(log\sqrt{g}\right)$. **[20 Marks]**

1993

144. Prove that the angular velocity or rotation at any point is equal to one half of the curl of the velocity vector V. **[20 Marks]**

145. Evaluate $\iint_S \Delta \times \vec{F}\vec{n} ds$ where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$ **[20 Marks]**

146. Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor or rank one **[20 Marks]**

1992

147. If $\vec{F}(x,y,z) = (y^2 + z^2)\vec{i} + (z^2 + x^2)\vec{j} + (x^2 y^2)\vec{k}$ then calculate $\int_c \vec{f}\,\overrightarrow{dx}$ where C consist of (i) The line segment form $(0,0,0)$ to $(1,1,1)$ (ii) the three line segments AB, BC and CD where A, B, C and D are respectively the points $(0,0,0), (1,0,0), (1,1,0)$ and $(1,1,1)$ (iii) the curve $\vec{x} + \overrightarrow{ui} + \overrightarrow{u^2 j} + \overrightarrow{u^2 k}, u$ from 0 to 1. **[20 Marks]**

148. If $\vec{a}$ and $\vec{b}$ are constant vectors, show that

$(i) div\left\{x \times (\vec{a} \times \vec{x})\right\} = -2\vec{x}\vec{a}$

$(ii) div\left\{x \times (\vec{a} \times \vec{x}) \times (\vec{b} \times \vec{x})\right\} = 2\vec{a}(\vec{b} \times \vec{x}) - 2b(\vec{a} \times \vec{x})$ **[20 Marks]**

149. Obtain the formula $div\vec{A} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^{-1}} \left\{ \left(\frac{g}{g_{ij}} \right)^{1/2} A(i) \right\}$ where $A(i)$ are physical components of $\vec{A}$ and use it to derive expression of $div\vec{A}$ in cylindrical polar coordinates **[20 Marks]**